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THE RANGE-ENERGY RELATION IN NUCLEAR EMULSIONS

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The use of large emulsion block detectors for the study of the masses and modes of decay of unstable particles has made it possible to measure the range of fairly energetic particles. In order to use such observations for good energy determinations an accurate knowledge of the relation between range and energy for different types of particles in nuclear emulsions upto fairly high energies is required. Energy values are needed for particles with masses equal to or greater than that of μ-mesons upto ranges of about 20 cm. Since range-energy relations for different singly charged particles can be directly transformed into one another provided their masses are known, it is sufficient to discuss the range-energy relation for protons. The required curve for protons should, therefore, extend to $20 \times \frac{M_p}{M_\mu} \sim 180$ cm. or an approximate proton energy of 2 Bev.

Experimentally the range-energy relation in nuclear emulsions has been investigated by various workers[1-4] upto proton energies of 40 MeV and a corresponding residual range of about 6 mm. In restricted energy intervals these results can be represented by empirical relations of the type $E = kR^x$ where E is the energy, and R the range of the particle. Experimental data for protons with energies between 40 MeV and 2 BeV could be obtained with particles accelerated in large synchro cyclotrons, but at present, the only point on the range-energy curve measured in this energy interval is that obtained by Heinz[5] for 342·5 MeV protons. Therefore, in the absence of more extensive detailed measurements, it becomes necessary to construct a range-energy relation which will represent all existing experimental data and permit reasonable interpolation and extrapolation.

In using the available experimental results one has to bear in mind the dependence of the stopping power of emulsions on their moisture content. When reduced to equal moisture content it is found that the experimental range-energy data for emulsions are related in a simple manner to the corresponding theoretical values for Aluminium and Lead as calculated by

Smith[6] and published in the form of graphs and tables by the Princeton Physics Department. It seems, then, that these theoretical calculations form a safe basis for reliable interpolation and extrapolation, though we hope that they can soon be replaced by accurate directly measured values from high energy accelerators. We nevertheless think it desirable to give in detail the range-energy curve which we are using in our work and describe the method by which it is constructed, because it enters into various mass determinations of hyperons and K-mesons which we have recently carried out.

PROCEDURE

The following experimental data on the range-energy relation of protons in Ilford emulsions are available:—

(*a*) Gibson, Prowse and Rotblat measured the range of proton tracks in the energy interval 2 to 21 MeV. They used Ilford C-2 emulsions of thickness 200 μ and 400 μ. After placing their emulsion into a vacuum for 6 to 12 hours in the presence of phosphorous pentoxide, they accept the manufacturer's density of 3·94 gm./cm.^{3}

(*b*) Heinz used protons of 342·5 MeV from the Berkeley cyclotron. He also uses C-2 Ilford emulsions and measures the emulsion density before and after exposure. His value for the emulsion density is 3·81 gm./cm.^{3}

(*c*) Bradner, Smith, Barkas and Bishop use protons in the energy interval 8 MeV to 40 MeV. They do not mention the density of the C-2 emulsions (100 μ thick) used in their experiment. They kept their emulsions in vacuum for six minutes but without any dehydrating agent (They did not find any difference when these plates were kept from six minutes upto six hours in vacuum).

We have normalised the measured values of proton ranges to an emulsion density of 3·94 gm./cm.^{3} The range given by Heinz for 342·5 MeV protons for example would (in terms of gm./cm.^{2}) be somewhat larger if 4·4% of water had been removed from his emulsion. We take the "normalised" range in Heinz's experiment to be 93·6 ± 0·25 gm./cm.^{2} equal to 23·73 ± ·063 cm. of emulsion. The data of Bradner *et al.* cannot be so normalised because in their paper the density of the emulsions used by them is not mentioned.

We now calculate, using the Princeton curves, the ratio K between the square of the range in emulsion (expressed in mm.) and the product of the ranges in Aluminium and Lead (expressed in gm./cm.^{2}). For the range in emulsion we use the data given by Gibson *et al.*, the normalised value of Heinz and the not-normalised values of Bradner *et al.* The results are shown in Plate VII, Fig. 1. It appears from this figure that:

(*a*) Bradner's values are only fairly consistent with each other and are not consistent with those of Gibson *et al.* Presumably they refer to emulsions of slightly varying density.

(*b*) The data of Gibson *et al.*, and the normalised point of Heinz indicate a value of K independent of energy and equal to $5{\cdot}075 \pm {\cdot}080$ (The fluctuations from the mean are of the same order as the errors involved in reading the Princeton curves).

It seems then, that the theoretical curves given by the Princeton Group, together with the assumption that the constant K is independent of energy, agree with experimental data and offer a rational procedure for interpolation and extrapolation. We have calculated various proton ranges in emulsion from the Princeton curves using $K = 5{\cdot}075$. They are given in Table I, and

TABLE I

Range of Protons in Nuclear Emulsions of Density 3·94 *gm./cm.*^{3}

E (MeV)	R (μ)	E (MeV)	R (mm.)
1	13·80	15	1·100
1·5	25 50	20	1·815
2	39·90	30	3·615
3	74·60	40	5·960
4	118·5	50	8·750
5	172·5	60	11·85
6	235·0	80	19·80
7	303·0	100	29·70
8	378·0	150	58·15
10	554·0	200	95·40
		300	192·0
		342·5	237·5
		400	296·5
		600	552·5
		800	831·0
		1000	1135·0

the corresponding curve is shown in Plate VIII, Fig. 2. Table I and Plate VIII, Fig. 2 refer to C-2 emulsions of density 3·94 gm./cm.^{3} Since the composition of C-2 emulsions is identical with that of G-5 emulsions they are applicable to G-5 emulsions as well. In addition to the range measurements available at various energies one accurately fixed point on the curve is obtained from μ-mesons of unique energy arising from the decay of π^+-meson at rest. Since the masses of π-meson and μ-meson are known within 0·2–0·4 electron masses[7] the μ-meson energy (4·12 MeV) is known accurately. The range R_0 of the μ-meson arising from the decay of π^+-meson at rest is $574 \cdot 5 \pm 7 \cdot 7\, \mu$ for emulsions of density 3·94 gm./cm.^{3} as read from the curve in Plate VIII, Fig. 2. The error in range reflects the errors in the π- and μ-meson masses. The range energy relation can be normalised to other G-5 or C-2 emulsions of varying moisture content by measuring the mean range R of flat μ-mesons, emitted from the decay of π^+-mesons at rest, and multiplying all observed ranges by the appropriate factor R/R_0.

Accurate measurements on the variation of the density of C-2 emulsions as a function of the relative humidity has been recently carried out by A. J. Oliver.[8] From these measurements, Barkas[9] has suggested that the density of 3·915 gm./cm.^{3} sometimes ascribed to Ilford emulsions at 50% relative humidity is too high. If, therefore, the density of 3·94 gm./cm.^{3} given by Gibson, Prowse and Rotblat to their emulsions is also too high, then we have to consider the good fit we obtained for Heinz's point on our curve as accidental. Even so, if our assumption that K is constant upto very high energies is correct, our range-energy relation will still give the correct shape to the curve; but will require proper renormalisation.

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EXPLANATION OF PLATES

PLATE VII

FIG. 1. The figure shows the experimentally determined ranges in nuclear emulsion for protons of various energies as observed by different investigators. The ordinate represents the square of the range in emulsion expressed in units of the geometric mean of the corresponding ranges in aluminium and lead. $K = \frac{R^2_{(Em)}}{R_{Al} \times R_{Pb}}$ (see text).

PLATE VIII

FIG. 2. The range of protons and π-mesons in nuclear emulsions is plotted against their kinetic energy (expressed in units of their respective rest energies).

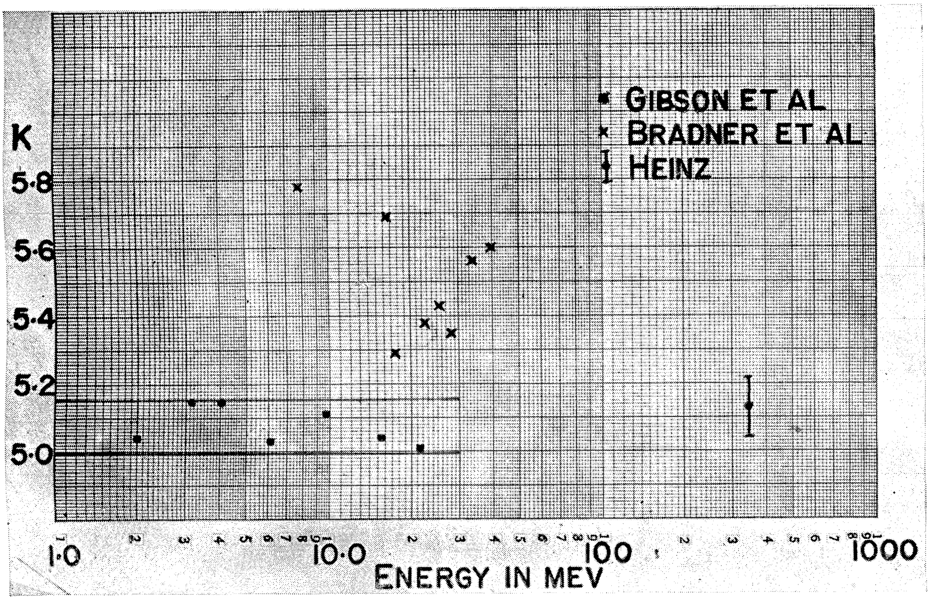
K
5·8
5·6
5·4
5·2
5·0
GIBSON ET AL
BRADNER ET AL
HEINZ
1·0
10·0
100
1000
ENERGY IN MEV

FIG. 1

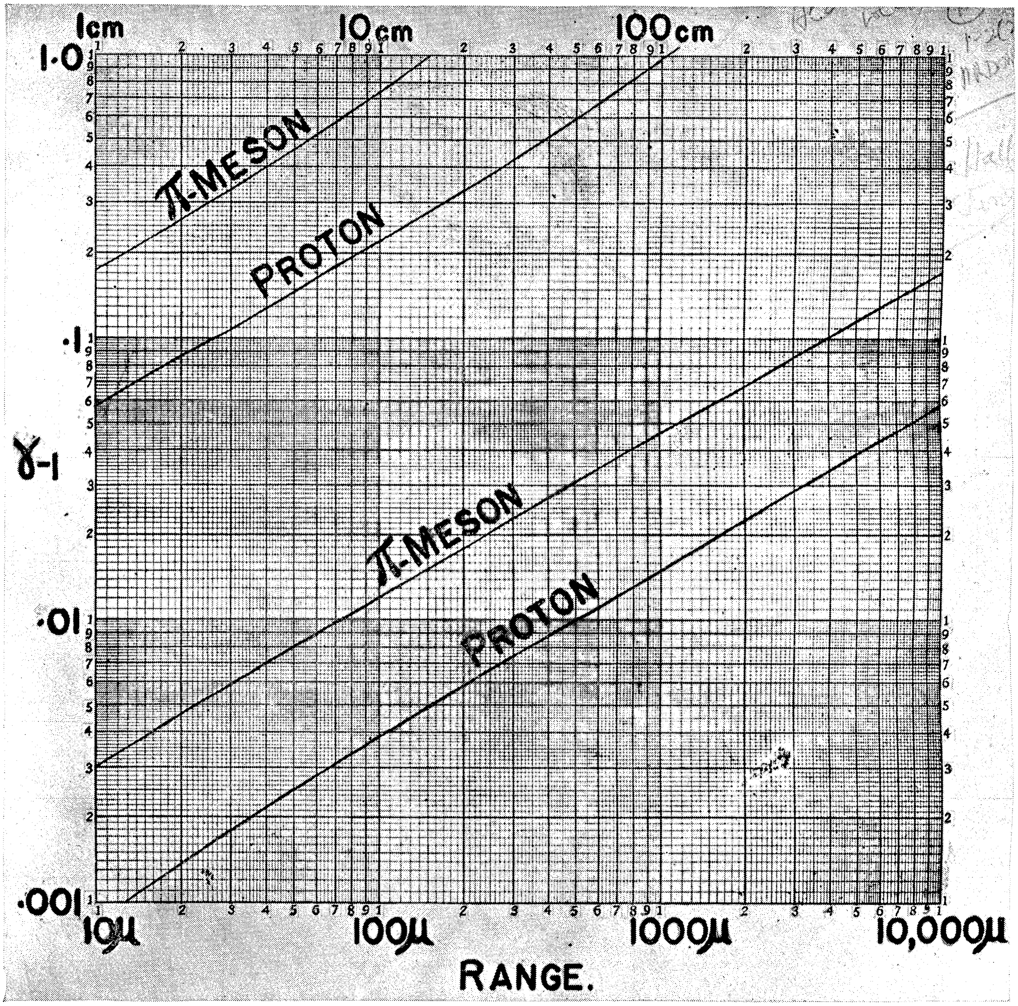
1cm
10cm
100cm
1·0
·1
·01
·001
γ-1
π-MESON
PROTON
π-MESON
PROTON
10μ
100μ
1000μ
10,000μ
RANGE.

FIG. 2