Reprinted from THE PHYSICAL REVIEW, Vol. 113, No. 3, 944–954, February 1, 1959
Printed in U. S. A.

V-A Theory and the Decay of the Λ Hyperon*

S. OKUBO AND R. E. MARSHAK, *University of Rochester, Rochester, New York*

AND

E. C. G. SUDARSHAN,† *University of Rochester, Rochester, New York, and Harvard University, Cambridge, Massachusetts*

(Received September 8, 1958)

The decay of the Λ hyperon is studied within the framework of the chirality-invariant four-fermion interaction. It is shown that the branching ratio of the charged and neutral modes, the s- to p-wave emission, as well as the sign and magnitude of the asymmetry parameter of the $p+\pi^-$ decay mode, can be understood on the basis of the *V-A* theory. Improvements upon the Born approximation, using dispersion theory, indicate that these conclusions are not invalidated by taking into account the pion-nucleon interaction.

1. INTRODUCTION

THE gross features of weak interactions (like coupling strengths and the breakdown of space reflection and charge conjugation invariances) are now fairly well understood.[1] Increasing confirmation has come from various experiments during the last few months for a four-fermion interaction in β decay of the *V-A* form. Indeed, the chirality-invariant *V-A* theory appears to be capable of explaining *all* experimental data on weak interactions involving nonstrange particles. We have already remarked[1] on the possibility of *all* weak interactions proceeding directly or indirectly, through a universal chiral four-fermion interaction with suitably chosen pairs of charged and neutral fields.

The *V-A* interaction finds its simplest expression[1–3] as a self coupling of a chiral current, composed additively of a lepton current (j^λ), a strangeness-conserving current of strongly interacting particles (J^λ) and a strangeness-nonconserving current of strongly interacting particles ($\mathcal{J}^\lambda$) in the form

$$\mathcal{L}_w=G\times\tfrac{1}{2}\{(j^\lambda+J^\lambda+\mathcal{J}^\lambda)^\dagger,\,(j_\lambda+J_\lambda+\mathcal{J}_\lambda)\}_+,$$

where G is the universal weak coupling constant. That the lepton currents j^λ involved in β decay and the decays of the muon, the pion, and the K meson are the positive chiral charge-exchange currents follows from the agreement of the experimental results with the predictions of this interaction. No such immediate connection exists between the form of the transition amplitude and the structure of the current operator for J^λ or $\mathcal{J}^\lambda$.

Consider for example, the matrix element for β decay: the transition amplitude is proportional to

$$\left\langle p,e\nu\left|\int d^3x\, J_\mu^\dagger(x)j^\mu(x)\right|n\right\rangle$$
$$=\int\left\langle p\left|\int d^3x\, J_\mu^\dagger(x)e^{ik\cdot x}\right|n\right\rangle \times\left\langle e\nu\left|\int d^3y\, j^\mu(y)e^{-ik\cdot y}\right|0\right\rangle\frac{d^3k}{(2\pi)^3}$$
$$=\left(\frac{m_pm_n}{E_pE_n}\right)^{\frac{1}{2}}\Lambda_\mu(p,n)\langle e\nu|\,j^\mu(k)\,|0\rangle.$$

Choosing the form

$$J_\mu=\bar\psi_n\gamma_\mu(1+\gamma_5)\psi_p,$$

one obtains

$$\Lambda_\mu=\bar u_p\gamma_\mu(f+g\gamma_5)u_n+\bar u_p\{\sigma^{\mu\nu}(h_1+h_2\gamma_5)+\delta^{\mu\nu}(h_3+h_4\gamma_5)\}(p_\nu-n_\nu)u_n,$$

where f, g, and h_i are invariant functions only of the momentum transfer at the np vertex. Notice that the terms involving h_i vanish in the limit of zero momentum-transfer. The two form factors f and g are the renormalization factors of the vector and axial vector parts of the current and are in general not equal (in the absence of renormalization effects $f=g=1$). The comparison of the lifetimes of the muon, O^{14} and the neutron,[4] give for the zero-momentum limits:

$$f(0)\simeq1,\quad g(0)\simeq1.2.$$

Thus, in spite of the strong interactions, the admixture of the negative chiral current of the physical nucleon states in the vertex function is small.

The absence thus far of any hyperon-lepton decays‡

* This work was supported in part by the U. S. Atomic Energy Commission. Some of the results contained in this paper were presented at the *1958 Annual International Conference on High-Energy Physics at CERN*, Proceedings edited by B. Ferretti (CERN, Geneva, 1958).

† On leave of absence from the Tata Institute of Fundamental Research, Bombay, India.

[1] E. C. G. Sudarshan and R. E. Marshak, *Proceedings of the Padua-Venice Conference, September, 1957;* Phys. Rev. **109**, 1860 (1958). See also, R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958), and J. J. Sakurai, Nuovo cimento **7**, 649 (1958).

[2] Okubo, Marshak, Sudarshan, Teutsch, and Weinberg, Phys. Rev. **112**, 665 (1958).

[3] E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958); the negative sign given in this paper refers to the nucleon asymmetry (the pion asymmetry has the opposite sign).

[4] Sosnovskii, Spivak, Prokofiev, Kutikov, and Dobrynin, referred to in the summary talk by M. Goldhaber, *Proceedings of the 1958 Annual International High-Energy Conference at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

‡ *Note added in proof.*—Two examples of the decay mode $\Lambda\rightarrow p+e+\nu$ have been observed by the Berkeley group; although no reliable branching ratio for this mode is thus far available, it appears reduced. See Crawford, *et al.*, Phys. Rev. Letters **1**, 377 (1958), and Nordin, *et al.*, Phys. Rev. Letters **1**, 380 (1958).

seems to indicate that the corresponding strangeness-violating vertex is strongly dependent upon the momentum transfer and much depressed with respect to the zero-energy value of the strangeness-conserving vertex. By virtue of this observation, the extension of the chiral coupling scheme to strangeness-violating currents $\mathcal{J}^\lambda$ is a specific hypothesis concerning the structure of the weak-interaction Lagrangian. It is one of the aims of the present paper to furnish empirical justification for this generalization.

The weak interactions thus fall into subcategories depending on the appropriate parts of the weak-interaction Lagrangian responsible for the transitions. The muon decay is the simplest, involving the self-coupling of the lepton current. Beta decay, pion decay, and muon capture involve the coupling of the lepton current j^λ with the strangeness-conserving current J^λ while the leptonic modes of the K mesons involve the coupling of j^λ to the strangeness-violating current $\mathcal{J}^\lambda$. Finally, one has the nonleptonic strangeness-violating decays which are presumed to proceed through the coupling of the two currents J^λ and $\mathcal{J}^\lambda$. The complications of the strong interactions appear in their full complexity in the last case.

This last category of slow processes differs in several essential respects from the leptonic modes. In view of the absence of a lepton current occurring as a factor in the interaction, it is not possible to express the transition amplitudes in terms of charge-exchange vector vertex operators. The corresponding vector vertex appearing in the leptonic modes is a far simpler object to deal with with regard both to selection rules and to renormalization properties. The consequences of specific assumptions about the isotopic spin transformation properties and equations of motion of the "currents" are thus best tested for the leptonic modes. For the nonleptonic modes, the transition amplitude, in general, bears no simple relation to the "current" structure of the interaction. For this reason, apart from the meagerness of the experimental data, there has been no incisive test of our hypothesis concerning the strangeness-violating decays. Furthermore, there is the ambiguity in deciding about the isotopic spin transformation properties and the individual fields entering in the corresponding current $\mathcal{J}^\lambda$. Among the strangeness-nonconserving weak decays, the most favorable case to consider at the present time is the decay of the Λ hyperon.

There are now at least three reasonably well-established pieces of information concerning the Λ decay: (1) The fraction of the Λ's decaying via the mode $p+\pi^-$ is 0.63 ± 0.03[5]; it is presumed that the remaining decays are via the mode $n+\pi^0$. (2) An up-down pion asymmetry $\epsilon=0.55\pm0.10$ has been observed[6] for the decay mode $p+\pi^-$, where $\epsilon=\alpha_- P_\Lambda$ with α_- the intrinsic pion asymmetry parameter and P_Λ the polarization of the decaying Λ. (3) The polarization of the decay proton from the unpolarized Λ has been observed to have a negative sign and a large magnitude[7] (of the order of 0.9). The last two experiments are fully concordant with each other and imply that the asymmetry parameter $\alpha_-\simeq+0.9$.[8]

The extension of the V-A theory to the Λ decay is rather unique and it is natural to take for the four-fermion interaction

$$H_w=G\bar{\psi}_p\gamma^\lambda(1+\gamma_5)\psi_\Lambda\cdot\bar{\psi}_n\gamma_\lambda(1+\gamma_5)\psi_p+\text{H.c.} \quad (1)$$

The isotopic spin transformation properties of the strangeness-violating current $\mathcal{J}^\lambda=\bar{\psi}_p\gamma^\lambda(1+\gamma_5)\psi_\Lambda$ are those of an isotopic spinor; we have shown elsewhere[2] that this choice is consistent with the K meson decays and that it predicts a value for the $K_2{}^0$ lifetime in close agreement with experiment. The choice of Λ hyperon alone is, of course, not dictated by the isospinor character of the $\mathcal{J}^\lambda$ current. For the present, however, we assume that $\mathcal{J}^\lambda$ consists only of (Λ,p) since this is such a simple choice; and it may turn out that the Λ hyperon is the most fundamental of the various hyperons. This is true, for example in the Sakata model and in the theory of Kobsarev and Okun[9] where the possibility of considering the n, p, Λ fields as the truly fundamental fields and identifying the other hyperons and mesons as composite structures[9] is discussed. In this paper we wish to discuss in greater detail than in a previous note[3] the consequences of considering the weak-interaction Hamiltonian (1) jointly with the strong pion-nucleon interaction.

2. BORN APPROXIMATION PREDICTIONS

We write down the transition matrix elements for the two decay modes of the hyperon as follows:

$$M_-\equiv M(\Lambda\to p+\pi^-)=\{(\tfrac{1}{3})^{\frac{1}{2}}A_3-(\tfrac{2}{3})^{\frac{1}{2}}A_1\}+\{(\tfrac{1}{3})^{\frac{1}{2}}B_3-(\tfrac{2}{3})^{\frac{1}{2}}B_1\}\boldsymbol{\sigma}\cdot\hat{k}, \quad (2)$$

$$M_0\equiv M(\Lambda\to n+\pi^0)=\{(\tfrac{2}{3})^{\frac{1}{2}}A_3+(\tfrac{1}{3})^{\frac{1}{2}}A_1\}+\{(\tfrac{2}{3})^{\frac{1}{2}}B_3+(\tfrac{1}{3})^{\frac{1}{2}}B_1\}\boldsymbol{\sigma}\cdot\hat{k}, \quad (3)$$

where $\hat{k}$ is the unit vector in the direction of the momentum of the pion in the rest system of the Λ, $\boldsymbol{\sigma}$ is the spin of the nucleon, and A_3 and B_3 (or A_1 and B_1) are

[5] Glaser, Good, and Morrison, *Proceedings of the 1958 Annual International High-Energy Conference at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

[6] Eisler, Plano, Samios, Schwartz, and Steinberger, Nuovo cimento **5**, 1700 (1957); Plano, Prodell, Samios, Schwartz, Steinberger, Bassi, Borelli, Puppi, Tanaka, Waloschek, Zoboli, Conversi, Franzini, Mannelli, Santangelo, Silvestrini, Glaser, Graves, and Perl, Phys. Rev. **108**, 1353 (1957); Crawford, Cresti, Good, Gottstein, Lyman, Solmitz, Stevenson, and Ticho, Phys. Rev. **108**, 1102 (1957).

[7] Boldt, Bridge, Caldwell, and Pal, Phys. Rev. Letters **1**, 256 (1958).

[8] Such a large asymmetry parameter is consistent only with spin ½ for the Λ hyperon; see T. D. Lee and C. N. Yang, Phys. Rev. **109**, 1755 (1958).

[9] S. Sakata, Progr. Theoret. Phys. **16**, 686 (1956); I. K. Kobsarev and L. B. Okun, *Proceedings of the 1958 Annual International High-Energy Conference at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

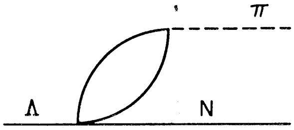

FIG. 1. Lowest order graph for Λ decay.

the s and p parts, respectively, of the transition matrix belonging to the total isotopic spin $I=\frac{3}{2}$ (or $I=\frac{1}{2}$) of the final pion-nucleon system. Now from the first experimental result cited above, it follows that

$$R\equiv|M_-|^2/|M_0|^2\simeq 2. \qquad (4)$$

Usually, (4) is explained by the selection rule $\Delta I=\frac{1}{2}$,[10] or equivalently in our notation, by assuming $A_3=B_3=0$.

It is desirable to investigate the most general condition under which (4) holds since it will turn out that the interaction (1) predicts (4) (in Born approximation) but nevertheless violates the $\Delta I=\frac{1}{2}$ selection rule. By using (2) and (3), (4) yields the following equality:

$$2\sqrt{2}\,\mathrm{Re}(A_1A_3^*+B_1B_3^*)=-(|A_3|^2+|B_3|^2). \qquad (5)$$

If we furthermore assume that the theory is invariant under time reversal, we can write

$$\begin{aligned} A_1&=\pm|A_1|e^{i\alpha_1}, \quad A_3=\pm|A_3|e^{i\alpha_3},\\ B_1&=\pm|B_1|e^{i\alpha_{11}}, \quad B_3=\pm|B_3|e^{i\alpha_{31}}, \end{aligned} \qquad (6)$$

where the α's are the usual pion-nucleon phase shifts at the final state energy $\simeq$37 Mev. Since the α's are quite small at this energy,[11] we may set them all equal to zero. This implies that the A's and B's are real numbers and (5) becomes

$$2\sqrt{2}(A_1A_3+B_1B_3)=-(A_3^2+B_3^2). \qquad (7)$$

One solution of (7) is $A_3=B_3=0$ ($\Delta I=\frac{1}{2}$ selection rule) but another possible solution is

$$A_3=-2\sqrt{2}A_1, \quad B_3=-2\sqrt{2}B_1. \qquad (8)$$

Let us next consider the asymmetry factors for the two decay modes of Λ. In terms of our notation, we can write

$$\alpha_-=2(A_3-\sqrt{2}A_1)(B_3-\sqrt{2}B_1)/[(\sqrt{2}A_1-A_3)^2+(\sqrt{2}B_1-B_3)^2], \qquad (9a)$$

$$\alpha_0=2(\sqrt{2}A_3+A_1)(\sqrt{2}B_3+B_1)/[(\sqrt{2}A_3+A_1)^2+(\sqrt{2}B_3+B_1)^2]. \qquad (9b)$$

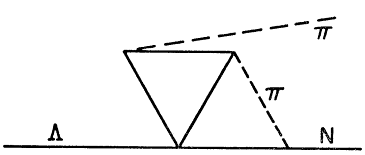

FIG. 2. A "triangle" diagram.

The ratio α of these asymmetry factors is given by

$$\alpha=\alpha_-/\alpha_0=(A_3-\sqrt{2}A_1)(B_3-\sqrt{2}B_1)/[2(\sqrt{2}A_3+A_1)(\sqrt{2}B_3+B_1)]. \qquad (10)$$

Obviously, when $\Delta I=\frac{1}{2}$, then $\alpha=1$; however, when (8) is true, we again have $\alpha=1$.

The interesting fact is that the chirality-invariant interaction (1) yields (8) in the lowest order (Born approximation). If we decompose (1) into tensor operators with respect to isotopic spin space, then we can rewrite (1) as follows:

$$H_w=H^{(\frac{3}{2})}+H^{(\frac{1}{2})}, \qquad (11)$$

where

$$H^{(\frac{3}{2})}=\tfrac{1}{3}G\{2\bar{\psi}_p\gamma_\mu(1+\gamma_5)\psi_\Lambda\cdot\bar{\psi}_n\gamma^\mu(1+\gamma_5)\psi_p-\bar{\psi}_n\gamma_\mu(1+\gamma_5)\psi_\Lambda\cdot\bar{\psi}_n\gamma^\mu(1+\gamma_5)\psi_n\}+\text{H.c.}, \qquad (12a)$$

$$H^{(\frac{1}{2})}=\tfrac{1}{3}G\{\bar{\psi}_p\gamma_\mu(1+\gamma_5)\psi_\Lambda\cdot\bar{\psi}_n\gamma^\mu(1+\gamma_5)\psi_p+\bar{\psi}_n\gamma_\mu(1+\gamma_5)\psi_\Lambda\cdot\bar{\psi}_n\gamma^\mu(1+\gamma_5)\psi_n\}+\text{H.c.} \qquad (12b)$$

In the lowest order of Λ-decay interaction (see Fig. 1), we can evaluate the A's and B's from (12a) and (12b). We can easily verify that we obtain (8). Thus, while the

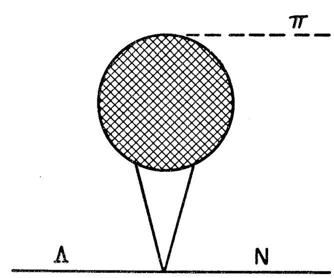

FIG. 3. The "bubble" black-box diagram.

chirality-invariant interaction (1) actually contributes a larger $\Delta I=\frac{3}{2}$ than $\Delta I=\frac{1}{2}$ matrix element to the decay, it gives the correct answer for the ratio of the $\Lambda\rightarrow p+\pi^-$ to $\Lambda\rightarrow n+\pi^0$ mode. In Born approximation, the V-A theory[3] also predicts that there is no transverse polarization and that $\alpha_-=+0.88$, which agrees both in sign and magnitude with experiment. The absolute transition probability, in Born approximation, diverges logarithmically. An estimate, using a cutoff of the order of the nucleon mass M (taking account of both the charged and neutral modes) leads to a lifetime $\sim 4\times 10^{-11}$ sec, too small by a factor of 10 compared to the experimental result. However, this is not serious, because Goldberger and Treiman[12] showed that the matrix element will be damped, if this loop is treated more rigorously.

3. IMPROVEMENTS ON THE BORN APPROXIMATION

The Born approximation calculations in the previous section led to results in agreement with experiment,

[10] See M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 407.

[11] A recent Rochester experiment at 41.5 Mev [Barnes, Rose, Giacomelli, Ring, and Miyake, Atomic Energy Commission Report NYO-2170 (to be published)] yields $\alpha_3=-0.1005\pm0.015$, $\alpha_{31}=-0.0477\pm0.0068$, $\alpha_1=0.1668\pm0.023$, $\alpha_{11}=-0.016\pm0.11$ (all numbers in radians).

[12] The application of dispersion relations to the decay of the pion was made by M. L. Goldberger and S. B. Treiman [Phys. Rev. **110**, 1178 (1958)].

except for some uncertainty in the absolute scale factor. However, since the pion-nucleon interaction is strong, one must investigate the effects of this interaction by taking into account the multiple scattering in the final state; also, one must consider Feynman diagrams more complicated than Fig. 1, leading to the decay. The multiple scattering couples the $p+\pi^-$ and $n+\pi^0$ modes and alters the phase relations of the $I=\frac{3}{2}$ and $I=\frac{1}{2}$ states so that one would expect the branching ratio and the asymmetry factors for the two modes to be changed.

Before we consider the multiple-scattering effects, we note that the next higher order correction to the Born approximation which is not a final-state interaction corresponds to a triangular loop (see Fig. 2). This "triangle" diagram (at least in lowest-order perturbation theory) no longer predicts the same ratio of the $p+\pi^-$ and $n+\pi^0$ modes as does the "bubble" diagram in Fig. 1, nor does it lead to a large asymmetry factor. It turns out that in lowest-order perturbation theory, the "triangle" diagram contributes much less than the "bubble" diagram and indeed, for the $p+\pi^-$ mode, the contribution vanishes identically due to mutual cancellation of various graphs. This latter result implies that

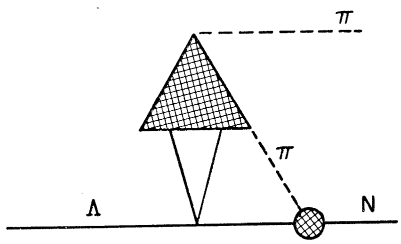

FIG. 4. The "triangle" black-box diagram.

a qualitative understanding of the large asymmetry factor in $p+\pi^-$ decay is possible, even without believing the magnitude of the contribution from the "triangle" diagram as given by perturbation theory. If the "triangle" contribution were large, it would reduce the asymmetry parameter for the mode $n+\pi^0$ and alter the branching ratio R. To the extent that the latter ratio is known to be nearly 2, it may be significant that perturbation theory predicts only a small contribution to the $n+\pi^0$ mode from this diagram.

We have indicated that some of the above results are largely independent of perturbation theory. In fact, using only charge independence of the pion-nucleon interaction, one can obtain the same predictions for the branching ratio R and the asymmetry factors α_0, α_- from the black-box diagram corresponding to the "bubble" diagram (see Fig. 3). Similarly, using charge independence and charge conjugation invariance of the strong interactions, one can deduce that the contribution to the $p+\pi^-$ mode from the black-box diagram (see Fig. 4), corresponding to the "triangle" diagram, vanishes. The proofs of these assertions follow.

To demonstrate that the total contribution from the "triangle" diagrams to the mode $\Lambda\rightarrow p+\pi^-$ vanishes,

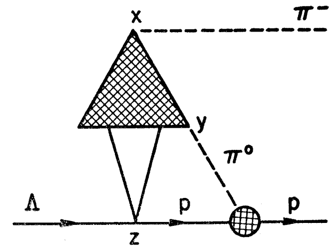

FIG. 5. "Triangle" black-box diagram involving virtual neutral pion.

consider the following vacuum expectation values of the time-ordered operators:

$$M_{\alpha\beta\gamma}{}^{\mu}=\langle 0|P(j_\alpha(x)j_\beta(y)J_\gamma{}^{\mu}(z))|0\rangle,$$
$$M_{\alpha\beta}{}^{\mu}=\langle 0|P(j_\alpha(x)j_\beta(y)J^{\mu}(z))|0\rangle,$$

where $j_\alpha(x)$ is the isotopic vector "current" to which the pions are coupled, so that

$$j_\alpha(x)=K_x\phi_\alpha(x),\quad K_x=\Box_x{}^2-\mu^2,$$

and $J_\alpha{}^\mu$, J^μ are defined in terms of the nucleon field operators in the form

$$J_\alpha{}^\mu(z)=\bar{\psi}(z)\gamma^\mu\tau_\alpha\psi(z);\quad J^\mu(z)=\bar{\psi}(z)\gamma^\mu\psi(z).$$

From charge independence, these should be of the forms

$$M_{\alpha\beta\gamma}{}^{\mu}\equiv\epsilon_{\alpha\beta\gamma}A^\mu,\quad M_{\alpha\beta}{}^{\mu}=\delta_{\alpha\beta}B^\mu;$$
$$(A^\mu=M_{123}{}^\mu,\ B^\mu=M_{11}{}^\mu).$$

From invariance under charge conjugation of the strong interactions, we obtain

$$B^\mu=-B^\mu=0.$$

Since, under charge conjugation,

$$J^\mu\rightarrow -J^\mu,\quad j_2\rightarrow -j_2,\quad j_{1,3}\rightarrow j_{1,3},$$

we get

$$M_{\alpha\beta}{}^{\mu}=0.$$

Now consider the complete matrix element for the decay $\Lambda\rightarrow p+\pi^-$ via the black-box diagrams in Figs. 5 and 6. The total transition amplitude is proportional to

$$\sqrt{2}\langle 0|P(j_+(x),j_3(y),\bar{\psi}_n(z)\gamma_\mu\psi_p(z))|0\rangle$$
$$+(\sqrt{2})^2\langle 0|P(j_+(x),j_-(y),\bar{\psi}_p\gamma_\mu\psi_p(z))|0\rangle$$
$$=\tfrac{1}{2}iA^\mu+2(\tfrac{1}{4}B^\mu-\tfrac{1}{4}iA^\mu)=\tfrac{1}{2}B^\mu=0.$$

However, this is not true for the $\Lambda\rightarrow n+\pi^0$ decay mode.

Similarly, for the simple black-box diagram (Fig. 3), the contributions of the charged and neutral modes are

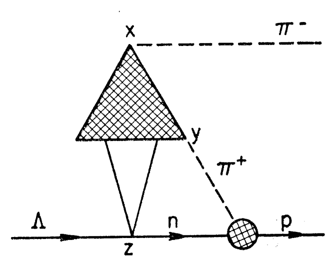

FIG. 6. "Triangle" black-box diagram involving virtual charged pion.

proportional to

$$\langle 0|P(\bar{\psi}_n(0)\gamma_\mu\psi_p(0),j_+(x))|0\rangle,\quad p+\pi^-$$
$$\langle 0|P(\bar{\psi}_p(0)\gamma_\mu\psi_p(0),j_3(x))|0\rangle,\quad n+\pi^0$$

and, again, by charge independence

$$\langle 0|P(\bar{\psi}(0)\gamma_\mu\psi(0),j_\alpha(x))|0\rangle=0,$$
$$\langle 0|P(\bar{\psi}(0)\gamma_\mu\tau_\alpha\psi(0),j_\beta(x))|0\rangle=\delta_{\alpha\beta}c^\mu,$$

so that the ratio of the charged to neutral amplitude is simply $\sqrt{2}$. We have thus proved our earlier statements.

4. DISPERSION-THEORETIC CONSIDERATIONS

In view of the remarks at the end of Sec. 3, it seems justified to attempt to treat in a rather rigorous way (using the methods of dispersion theory[12]) the multiple-scattering corrections to the "bubble" diagram. The starting point is the expression for the S matrix element for the decay $\Lambda\rightarrow N+\pi$ in terms of the Heisenberg operators of the baryon and pion fields. With the weak interaction considered only in first order, the invariant transition matrix element may be written[13]

$$\begin{aligned}T(p,k,q)&=i(2\pi)^{9/2}(2k_0)^{\frac{1}{2}}(p_0/m)^{\frac{1}{2}}(q_0/m_\Lambda)^{\frac{1}{2}}\langle N\pi|H_w(0)|\Lambda\rangle\\&=-(2\pi)^3(2k_0)^{\frac{1}{2}}(q_0/m_\Lambda)^{\frac{1}{2}}\bar{u}_N(p)\\&\quad\times\int d^4x\, e^{-ip\cdot x}D_x\langle\pi|P(\psi(x)H_w(0))|\Lambda\rangle.\end{aligned}\tag{13}$$

Here p, q, and k are the 4-momenta of the nucleon, hyperon, and pion and m, m_Λ are the nucleon and hyperon masses, respectively. H_w is the weak interaction (1) and

$$D_x=(\gamma^\mu\partial/\partial x_\mu+m).$$

From the covariance of the theory, $T(pkq)$ can be written in the form (I is the isotopic spin of the $N\pi$ system):

$$\bar{u}_N(p)\{(M_1{}^I+M_2{}^I\gamma_5)+i\gamma k(M_3{}^I+M_4{}^I\gamma_5)\}u_\Lambda(q),$$

which can be reduced to the form

$$\bar{u}_N(p)\{F_1{}^I(m_\Lambda)+F_2{}^I(m_\Lambda)\gamma_5\}u_\Lambda(q),\tag{14}$$

by using the Dirac equations for p and Λ, where $F_1{}^I$ and $F_2{}^I$ are related to A_I and B_I in Sec. 2 in a simple way.

The construction of the dispersion relation follows in the standard fashion by noting that the matrix element (13) can be re-expressed in terms of a "Born" amplitude (coming from an equal-time commutator; see below) and a causal amplitude. Provided that the functions are well-behaved, the real ("dispersive") and imaginary ("absorptive") parts of this causal amplitude are Hilbert transforms of each other. On the other hand, the absorptive part can be expanded in terms of a complete set of intermediate states. To make the problem tractable, it is necessary to assume that only one-pion states contribute to the explicit sum-over-intermediate-states expression for the absorptive amplitude; and that the Λ-hyperon field may be treated in perturbation. The inclusion of the Λ-hyperon interactions exactly leads to a complicated integral relation between the decay amplitude and several other matrix elements of the weak interaction, which reflect the possibility of having a cloud of virtual particles in the proper field of the Λ when it decays; this corresponds to Feynman diagrams with irreducible vertex modifications involving the incoming line. Similarly, the inclusion of other possible intermediate states (say, two-meson states) generates a term in the integral equation which involves a pion-production matrix element as well as the matrix element for the virtual two-pion Λ decay

$$\Lambda\rightarrow N+\pi+\pi.$$

This same amplitude can be related back to the one-pion decay, thus leading to two coupled integral equations for these amplitudes. In omitting consideration of these two-meson states, we fail to include, among other things, the contribution from the "triangle" diagrams. Since the contribution from the "triangle" diagram (Figs. 5 and 6) for the $p+\pi^-$ decay vanishes, we feel justified in neglecting the two-pion intermediate states.

We are therefore calculating diagrams of the type shown in Fig. 7. Explicitly carrying out the differentiation implicit in D_x in Eq. (13), we obtain

$$\begin{aligned}T(p,k,q)=&-(2\pi)^3(2k_0)^{\frac{1}{2}}(q_0/m_\Lambda)^{\frac{1}{2}}\bar{u}_p(p)\int d^4x\, e^{-ip\cdot x}\\&\times\left\langle\pi^-\left|\left\{\theta(x_0)[J_p(x),H_w(0)]\right.\right.\right.\\&\left.\left.\left.+\frac{1}{i}\gamma_4\delta(x_0)[\psi_p(x),H_w(0)]\right\}\right|\Lambda\right\rangle,\end{aligned}\tag{15}$$

where $J_p(x)$ is defined by

$$\left(\gamma^\mu\frac{\partial}{\partial x_\mu}+m\right)\psi_p(x)=J_p(x),\tag{16}$$

and where we have made use of the (known) mass spectrum to replace $P(J_p(x)H_w(0))$ by $\theta(x_0)[J_p(x),H_w(0)]$ in the expression for the matrix element. We can now decompose $T(p,k,q)$ in the form

$$T(p,k,q)=T^{(B)}(p,k,q)+D(p,k,q)+iA(p,k,q),\tag{17}$$

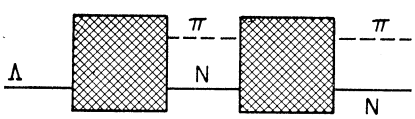

FIG. 7. The strong interactions considered in the present theory.

[13] Lehmann, Symanzik, and Zimmermann, Nuovo cimento 1, 205 (1955).

where the "Born" amplitude is

$$T^{(B)}(p,k,q)=-(2\pi)^3(2k_0)^{\frac{1}{2}}(q_0/m_\Lambda)^{\frac{1}{2}}\bar{u}_p(p)\int d^4x\, e^{-ip\cdot x}$$
$$\times\left\langle\pi^-\left|\frac{1}{i}\gamma_4\delta(x_0)[\psi_p(x),H_w(0)]\right|\Lambda\right\rangle \quad (18a)$$

while the dispersive (D) and absorptive (A) amplitudes are

$$D(p,k,q)=-(2\pi)^3(2k_0)^{\frac{1}{2}}(q_0/m_\Lambda)^{\frac{1}{2}}\bar{u}_p(p)\int d^4x\, e^{-ip\cdot x}$$
$$\times\langle\pi^-|\tfrac{1}{2}\epsilon(x_0)[J_p(x),H_w(0)]|\Lambda\rangle, \quad (18b)$$

$$A(p,k,q)=-(2\pi)^3(2k_0)^{\frac{1}{2}}(q_0/m_\Lambda)^{\frac{1}{2}}\bar{u}_p(p)\int d^4x\, e^{-ip\cdot x}$$
$$\times\left\langle\pi^-\left|\frac{1}{2i}[J_p(x),H_w(0)]\right|\Lambda\right\rangle. \quad (18c)$$

It must be stressed that our Born term $T^{(B)}$ comes from the equal-time commutator which is different from the usual case where it originates from the discrete terms. From covariance only, one can explicitly write down the matrix dependence in the form

$$T(p,k,q)=\bar{u}_p(p)\{(M_1+M_2\gamma_5)+i\gamma\cdot k(M_3+M_4\gamma_5)\}u_\Lambda(q),$$

where M_i are invariant functions; similar expressions in terms of M_i^B, D_i, and A_i can be written down for $T^{(B)}$, D, and A. If we now explicitly use the invariance of H_w and of the strong interactions under time reversal, one may show that all the M_i^B, D_i, and A_i are real. While the physical amplitude is defined only on the mass shell with

$$-p^2=m^2,\quad -q^2=m_\Lambda^2,\quad -k^2=m_\pi^2,\quad p+k=q,$$

the quantities introduced by Eq. (4) may be defined for arbitrary values of the momenta, especially without the last condition.

We now assume that the Λ hyperon is totally uncoupled and may hence be treated in perturbation. It is then obvious that the invariant functions M_i, D_i, and A_i are independent of q and depend only on the invariant $p\cdot k$. By virtue of the causal properties of the transition amplitude, T, M_i, D_i, and A_i are then analytic functions of the variable $\nu=-p\cdot k$ and consequently we can write dispersion relations of the form

$$M_i(\nu)=M_i^{(B)}(\nu)+\frac{1}{\pi}\int_{-\infty}^{\infty}d\nu'\frac{A_i(\nu')}{\nu'-\nu-i\epsilon}, \quad (19)$$

where we have used the relation

$$\mathrm{Im}\{M_i-M_i^{(B)}\}=A_i.$$

This last relation follows from time-reversal invariance.

To calculate the right-hand side of Eq. (19) we express $A(p,k,q)$ [using (18c)] in terms of a complete set of states $|n\rangle$ in the form

$$A(p,k,q)=-(2\pi)^{\frac{3}{2}}(2k_0)^{\frac{1}{2}}\frac{(2\pi)^4}{2i}\bar{u}_p(p)$$
$$\times\sum_n\{\langle\pi^-|J_p(0)|n\rangle\langle n|\bar{J}_\Lambda(0)|0\rangle\delta(n-p-k)$$
$$+\langle\pi^-|\bar{J}_\Lambda(0)|n\rangle$$
$$\times\langle n|J_p(0)|0\rangle\delta(p+n)\}u_\Lambda(q), \quad (20)$$

where $\bar{J}_\Lambda(x)=[\delta/\delta\psi_\Lambda(x)]H_w(x)$ and Λ was treated in perturbation; the summation over n is to be understood as the average of the sum over incoming states and over outgoing states in order to preserve the reality conditions on the amplitudes, independent of approximation.[12] The second term on the right-hand side of Eq. (20) gives zero for $-n^2=-p^2=m^2$ and thus n must be a one-antinucleon state and we are left with only the first term since $(n|J_p(0)|0)=0$ in this case. The lowest state is an isolated state with $|n_0\rangle$=neutron and from covariance alone, we may put

$$\langle n_0|\bar{J}_\Lambda(0)|0\rangle=-(2\pi)^{-\frac{3}{2}}(m/n_0)^{\frac{1}{2}}\bar{u}(n_0)(c_4+c_3\gamma_5),$$

where c_3 and c_4 are pure numbers; the contributions to $A_i(\nu)$ become

$$A_1^{(n_0)}(\nu)=0,\quad A_2^{(n_0)}(\nu)=0,$$
$$A_3^{(n_0)}(\nu)=(\pi/\sqrt{2})g\delta(\nu+\tfrac{1}{2}\mu^2)c_3,$$
$$A_4^{(n_0)}(\nu)=(\pi/\sqrt{2})g\delta(\nu+\tfrac{1}{2}\mu^2)c_4.$$

The contribution to the transition amplitude can now be calculated and corresponds to the diagram shown in Fig. 8 which represents a self-mass term nondiagonal in strangeness and divergent in general. One may add a counter term[14] to the original interaction to cancel this divergence but such a subtraction leaves ambiguous finite terms (because of the different masses of Λ and the nucleon). We shall omit any explicit mention of this term. If there exist any finite contributions, they may be absorbed into the Born amplitude.

The next set $|n\rangle$ consists of states containing one meson and one nucleon. First, take the contribution from the $|p\pi^-\rangle$ states. Then the matrix element $\langle p\pi^-|\bar{J}_\Lambda(0)|0\rangle$ is proportional to the transition amplitude we are studying in which Λ is treated by perturbation while the matrix element $\langle\pi^-|J_p(0)|p\pi^-\rangle$ is related

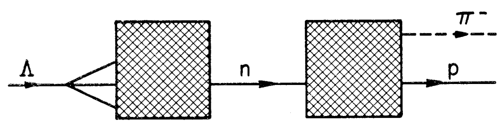

FIG. 8. The self-mass diagram.

[14] See S. Weinberg, Phys. Rev. **106**, 1301 (1957).

to the pion-proton scattering amplitude. In fact

$$\langle\pi^-(k)|J_p(0)|p_{(p')}\pi^-(k')\rangle$$
$$=(2\pi)^{-9/2}(2k_0)^{-\frac{1}{2}}(2k_0')^{-\frac{1}{2}}(m/p_0')^{\frac{1}{2}}\times\left\{(A^{(+)}+A^{(-)})^*-i\gamma\cdot\frac{k+k'}{2}(B^{(+)}+B^{(-)})^*\right\}u_p(p'),\quad(21)$$

where $A^{(\pm)}$, $B^{(\pm)}$ are the quantities defined by Chew, Goldberger, Low, and Nambu[15] and can be expressed in terms of the pion-nucleon scattering phase shifts α_{IJ}. Summing over the intermediate proton spins and momenta, one obtains the total contribution for $p+\pi^-$ intermediate states:

$$A^{(p+\pi^-)}(p,k,q)=\frac{1}{8\pi^2}\int\int d^4k'd^4p'\,\theta(k_0')\theta(p_0')\delta(k'^2+\mu^2)\delta(p'^2+m^2)\times\delta^{(4)}(p+k-p'-k')\bar{u}_p(p)\left\{A^*-i\gamma\cdot\frac{k+k'}{2}B^*\right\}\times(m-i\gamma\cdot p')\{[M_1(\nu')+M_2(\nu')\gamma_5]+i\gamma\cdot k'[M_3(\nu')+M_4(\nu')\gamma_5]\}u_\Lambda(q).\quad(22)$$

Upon performing the p' integration and rearranging the γ matrices, this can be written in the form

$$A_i^{(p+\pi^-)}(p\cdot k)=-\frac{1}{16\pi^2}\int d^4k'\,\theta(k_0')\theta(p_0+k_0-k_0')\delta(k'^2+\mu^2)\times\delta[\mu^2-p\cdot k+k'\cdot(p+k)]\,\mathrm{Re}G_i(p',k',p,k),\quad(23)$$

where

$$G_1=-A^*M_1m\left[2-\frac{\mu^2(p\cdot k')-\nu(k\cdot k')}{\nu^2-m^2\mu^2}\right]-A^*M_3\left[\mu^2\left\{1-\frac{m^2(k\cdot k')-\nu(p\cdot k')}{\nu^2-m^2\mu^2}\right\}-2(\nu^2+m^2)\left\{\frac{\mu^2(p\cdot k')-\nu(k\cdot k')}{\nu^2-m^2\mu^2}\right\}\right]+B^*M_1\left[\nu\frac{\mu^2(p\cdot k')-\nu(k\cdot k')}{\nu^2-m^2\mu^2}-(2\nu+\mu^2+k\cdot k')\right]+B^*M_3m(\mu^2+2\nu)\left[\frac{\mu^2(p\cdot k')-\nu(k\cdot k')}{\nu^2-m^2\mu^2}\right],\quad(24a)$$

$$G_2=A^*M_1\left[1-\frac{m^2(k\cdot k')-\nu(p\cdot k')}{\nu^2-m^2\mu^2}\right]+B^*M_1m\left[\frac{\mu^2(p\cdot k')-\nu(k\cdot k')}{\nu^2-m^2\mu^2}\right]+A^*M_3m\left[\frac{\mu^2(p\cdot k')-\nu(k\cdot k')}{\nu^2-m^2\mu^2}-2\frac{m^2(k\cdot k')-\nu(p\cdot k')}{\nu^2-m^2\mu^2}\right]+B^*M_3\left[\mu^2-(2\nu+\mu^2)\frac{m^2(k\cdot k')-\nu(p\cdot k')}{\nu^2-m^2\mu^2}\right].\quad(24b)$$

The expression for $G_2(G_4)$ is obtained from $G_1(G_3)$ by replacing M_1 by M_2 and M_3 by M_4. To connect these expressions to the phase shifts, we recall the definitions of $A^{(\pm)}$, $B^{(\pm)}$.[15] Taking the barycentric system of p and k and doing the angle integrations, we obtain, after a series of tedious but straightforward reductions,

$$A_1(p\cdot k)=\tfrac{1}{2}\frac{k}{\omega}\,\mathrm{Re}[\{[\omega+m]f_{0+}^*+[\omega-m]f_{1-}^*\}M_1+(\omega^2-m^2)[f_{1-}^*-f_{0+}^*]M_3],\quad(25a)$$

$$A_3(p\cdot k)=\tfrac{1}{2}\frac{k}{\omega}\,\mathrm{Re}[(f_{1-}^*-f_{0+}^*)M_1+\{[\omega-m]f_{0+}^*+[\omega+m]f_{1-}^*\}M_3],\quad(25b)$$

where $\omega=[-(p+k)^2]^{\frac{1}{2}}$ is the total energy of the pion and the nucleon in the barycentric system and $f_{l\pm}$ is related to the phase shift $\alpha_{l\pm}$ by the equation

$$f_{l\pm}=\frac{1}{k}\exp(i\alpha_{l\pm})\sin\alpha_{l\pm}\quad(26)$$

for a pure isotopic spin state.

In exactly similar fashion, we may calculate the contribution from the $n+\pi^0$ intermediate states. These contributions are related to the decay amplitude $\Lambda\to n+\pi^0$ and the charge-exchange scattering amplitude. Thus we obtain an integral equation coupling the $\Lambda\to p+\pi^-$ amplitude to itself and to the $\Lambda\to n+\pi^0$ amplitude and involving the direct and charge-exchange pion-nucleon scattering amplitudes. These coupled amplitudes separate if we decompose according to the isotopic spin. The dispersion integrals now assume the form

$$M_1^I(\omega)-M_1^{I(B)}(\omega)=\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\frac{k'}{\omega'^2-\omega^2-i\epsilon}\mathrm{Re}\{(\omega'+m)f_{0+}^{I*}\times[M_1^I(\omega')-(\omega'-m)M_3^I(\omega')]+(\omega'-m)\times f_{1-}^{I*}[M_1^I(\omega')+(\omega'+m)M_3^I(\omega')]\},\quad(27a)$$

[15] Chew, Goldberger, Low, and Nambu, Phys. Rev. 106, 1337 (1957).

$$M_3{}^I(\omega)-M_3{}^{I(B)}(\omega)$$
$$=\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\frac{k'}{\omega'^2-\omega^2-i\epsilon}$$
$$\times\mathrm{Re}\{-f_{0+}{}^{I*}[M_1{}^I(\omega')-(\omega'-m)M_3{}^I(\omega')]$$
$$+f_{1-}{}^{I*}[M_1{}^I(\omega')+(\omega'+m)M_3{}^I(\omega')]\},\quad(27\text{b})$$

with corresponding expressions for M_2, M_4. We may now define new amplitudes $F_1{}^I(\omega)$, $F_2{}^I(\omega)$ by

$$F_1{}^I(\omega)=M_1{}^I(\omega)-(\omega-m)M_3{}^I(\omega),\quad(28\text{a})$$
$$F_2{}^I(\omega)=M_1{}^I(\omega)+(\omega+m)M_3{}^I(\omega),\quad(28\text{b})$$

and rewrite the integral equations in terms of F_i: for $I=\frac{3}{2}$, these take the form

$$F_1{}^{\frac{3}{2}}(\omega)-F_1{}^{\frac{3}{2}(B)}(\omega)$$
$$=\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left\{\frac{\mathrm{Re}[F_1{}^{\frac{3}{2}}(\omega')\varphi_3{}^*(\omega')]}{\omega'-\omega-i\epsilon}+\frac{\mathrm{Re}[F_2{}^{\frac{3}{2}}(\omega')\varphi_{31}{}^*(\omega')]}{\omega'+\omega}\right\},\quad(29\text{a})$$
$$F_2{}^{\frac{3}{2}}(\omega)-F_2{}^{\frac{3}{2}(B)}(\omega)$$
$$=\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left\{\frac{\mathrm{Re}[F_1{}^{\frac{3}{2}}(\omega')\varphi_3{}^*(\omega')]}{\omega'+\omega}+\frac{\mathrm{Re}[F_2{}^{\frac{3}{2}}(\omega')\varphi_{31}{}^*(\omega')]}{\omega'-\omega-i\epsilon}\right\},\quad(29\text{b})$$

where we have put

$$\varphi_3(\omega)=kf_{0+}{}^{(\frac{3}{2})}(k)=e^{i\alpha_3}\sin\alpha_3,\quad(30\text{a})$$
$$\varphi_{31}(\omega)=kf_{1-}{}^{(\frac{3}{2})}(k)=e^{i\alpha_{31}}\sin\alpha_{31}.\quad(30\text{b})$$

For $I=\frac{1}{2}$, we have only to replace φ_3 and φ_{31} by φ_1 and φ_{11}, respectively. We see, therefore, that in the one-meson approximation, the physical transition amplitude satisfies a singular integral equation.

So far, we have not specified H_w. If we choose the expression (1), we get

$$T^{(B)}=-(2\pi)^{\frac{3}{2}}(2k_0)^{\frac{1}{2}}\left\langle\pi^-\left|\frac{G}{i}\bar{\psi}_n(0)\gamma_\mu(1+\gamma_5)\psi_p(0)\right|0\right\rangle$$
$$\times\bar{u}_p(p)\gamma_\mu(1+\gamma_5)u_\Lambda(q).\quad(31)$$

Hence

$$M_1{}^{(B)}=M_2{}^{(B)}=0,\quad M_3{}^{(B)}=M_4{}^{(B)}=f,$$

where f is a constant (whose value may be computed by the more detailed analysis of Goldberger and Treiman[12]). These Born amplitudes coincide with the lowest-order Born approximations discussed in the text. (Recall however that the residual single nucleon terms, if any, are also to be absorbed into this "Born" term.) We may now put

$$F_1{}^{I(B)}=-(\omega-m)f^I,\quad F_2{}^{I(B)}=(\omega+m)f^I,$$
$$f^{\frac{3}{2}}=-2\sqrt{2}f^{\frac{1}{2}}=f.\quad(32)$$

The solution of the singular integral equations (29) can be obtained using the techniques originally developed by Muskhelishvili[16] (the details are given in the Appendix):

$$F_1{}^{\frac{3}{2}}(z)=f\{m-z+\phi^{(\frac{3}{2})}\}\exp[K^{\frac{3}{2}}(z)+i\Delta_3(z)],$$
$$F_2{}^{\frac{3}{2}}(z)=f\{m+z+\phi^{(\frac{3}{2})}\}\exp[K^{\frac{3}{2}}(-z)+i\Delta_{31}(z)],$$
$$F_1{}^{\frac{1}{2}}(z)=-\frac{f}{2\sqrt{2}}\{m-z+\phi^{(\frac{1}{2})}\}\exp[K^{\frac{1}{2}}(z)+i\Delta_1(z)],\quad(33\text{a})$$
$$F_2{}^{\frac{1}{2}}(z)=-\frac{f}{2\sqrt{2}}\{m+z+\phi^{(\frac{1}{2})}\}\exp[K^{\frac{1}{2}}(-z)+i\Delta_{11}(z)],$$
$$(z\geq m+\mu),$$

where

$$\phi^{(\frac{3}{2})}=\frac{\mathcal{P}}{\pi}\int_{m+\mu}^{\infty}d\omega\{\sin\Delta_{31}(\omega)\times\exp[K^{\frac{3}{2}}(-\omega)]-\sin\Delta_3(\omega)\exp[K^{\frac{3}{2}}(\omega)]\},$$
$$\phi^{(\frac{1}{2})}=\frac{\mathcal{P}}{\pi}\int_{m+\mu}^{\infty}d\omega\{\sin\Delta_{11}(\omega)\times\exp[K^{\frac{1}{2}}(-\omega)]-\sin\Delta_1(\omega)\exp[K^{\frac{1}{2}}(\omega)]\},\quad(33\text{b})$$
$$K^{\frac{3}{2}}(z)=\frac{\mathcal{P}}{\pi}\int_{m+\mu}^{\infty}d\omega\left[\frac{\Delta_3(\omega)}{\omega-z}+\frac{\Delta_{31}(\omega)}{\omega+z}\right],$$
$$K^{\frac{1}{2}}(z)=\frac{\mathcal{P}}{\pi}\int_{m+\mu}^{\infty}d\omega\left[\frac{\Delta_1(\omega)}{\omega-z}+\frac{\Delta_{11}(\omega)}{\omega+z}\right],$$

and $\Delta(\omega)$ is related to the scattering phase shifts $\alpha(\omega)$:

$$\Delta_\gamma(z)=\frac{1}{2i}\ln\left[\frac{1+i\exp(i\alpha_\gamma)\sin\alpha_\gamma}{1-i\exp(-i\alpha_\gamma{}^*)\sin\alpha_\gamma{}^*}\right];$$
$$\gamma=3,31,1,11.\quad(33\text{c})$$

From the expressions (33a), (33b), (33c), we can in principle evaluate the effect of the pion-nucleon interaction exactly. However, the phase shifts α_3, etc. are not known beyond about 300 Mev and even up to these energies the empirically determined values are very unreliable. Thus, to assess the effect of the pion-nucleon interaction, we are forced to extrapolate the phase shifts. In this connection, it is important to notice that since the spin of the Λ hyperon is $\frac{1}{2}$, only the scattering in the $J=\frac{1}{2}$ states ($s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$) are relevant, and one recalls that these phase shifts are small in the region

[16] See S. G. Mikhlin, *Integral Equations* (Pergamon Press, London, 1957).

TABLE I. Comparison of the dispersion theory predictions with the Born approximation results. α is the intrinsic (pion) asymmetry parameter, x is the ratio of the p- to s-wave amplitudes, and R is the ratio of the decay rate via the charged mode to the decay rate via the neutral mode.

Parameter	Born approximation	Dispersion theory (A)
α_-	$+0.88$	$+0.97$
α_0	$+0.88$	$+0.91$
R	2.0	2.15
x_-	0.60	$0.82+0.14i$
x_0	0.60	$1.14+0.46i$

where they are known and that they do not appear to possess any resonances even at the higher energies. In the vicinity of 300 Mev, the s-wave phase shifts α_3, α_1 are $\approx 20°$ and α_1 approaches α_3; α_{31} and α_{11} are much smaller.[17] A typical extrapolation of the phase shifts, which exaggerates[18] the effect of the pion-nucleon interaction (putting the values at higher energy equal to the values at 300 Mev lab energy) leads to the results given as (A) in Table I. It should be stressed that the estimate is extremely crude and only the qualitative features of these predictions should be considered significant. In Table I, x is the ratio of the p- to s-wave amplitude; thus the effect on x of the pion-nucleon interaction can be quite large although the extrapolation gives only a crude upper limit. It is, however, interesting to note that the presently known experimental parameters, α_- and R for the decay of the Λ are rather insensitive to these large modifications.

It should be mentioned that some information concerning the ratio x comes from the ratio of the mesonic to the nonmesonic decay mode of hyperfragments, which seems to be inconsistent with a value of $|x|$ larger than unity.[19] However, one should bear in mind that there are at present no data directly available on the parameter x in free Λ decay. The transverse polarization of the resulting nucleon in Λ decay or the correlation of the Λ spin and the proton spin depends on the quantity $(1-|x|^2)$ and a measurement of either of these would provide a sensitive test for $|x|$. We have already remarked that our extrapolation of the phase shifts probably overestimates the pion-nucleon interaction effects. For example, if we equate all phase shifts to zero beyond 300 Mev, the $|x_-|$ and $|x_0|$ are greatly reduced and are given below as (B) in Table II.

A different type of question one has to consider concerns the effect of adding terms to H_w [in addition to those considered in (1)]. We noticed earlier in our discussion that even though the over-all selection rule $\Delta I=\frac{1}{2}$ is not obeyed by (1), it nevertheless leads to a ratio $R=2$. With terms added to H_w this result no

TABLE II. Comparison of two extrapolations.

Parameter	Dispersion theory A	Dispersion theory B
$\lvert x_-\rvert$	0.83	0.66
$\lvert x_0\rvert$	1.34	0.96

longer obtains; in particular, if we replace the current $\bar{\psi}_n\gamma^\mu(1+\gamma_5)\psi_p$ by terms involving Ξ or Σ hyperons,[3] then in lowest order, these interactions contribute only to the charged decay mode of the Λ. Since the weak interactions are treated only to lowest order, the contributions of the interactions involving Ξ or Σ are additive (see Fig. 9). The expected final branching ratio would then depend on the relative magnitudes and more particularly on the relative phases of the different contributions. We shall omit any such complicated considerations and merely note that in the framework of the dispersion theory treatment, these complications do not affect the relative s and p Born amplitudes; hence they are numerically irrelevant in determining x_0, x_-, and consequently the asymmetry parameter α_-; one predicts a large asymmetry parameter in any case, depending essentially only on the (known) pion-baryon coupling.

5. DISCUSSION

Our investigation of the Λ decay thus brings out the following points. The expression (1) for the weak decay leads not only to a qualitative but to a good quantitative understanding of the known features of this decay, despite the abandonment of the old $\Delta I=\frac{1}{2}$ selection rule. The surprising success of the Born calculations is explained by the fact that the branching ratio and the asymmetry parameter α_- are not particularly sensitive to the ratio of s and p amplitudes which is substantially altered by the improvements discussed in this paper. We have seen that the higher-order diagrams discriminate between the charged and neutral modes of decay and the possibility is indicated that the branching ratio may be unaffected while the asymmetry parameter α_0 is reduced below α_-; measurement of the ratio α_-/α_0 would be most interesting since the old $\Delta I=\frac{1}{2}$ selection rule unequivocally predicts unity for this ratio whereas our theory suggests a somewhat larger value.

The apparent success of the Born approximation for the decay tempts one to speculate that the factorization of the decay matrix element in the form

$$\langle p\pi^-|\mathcal{J}_\lambda{}^\dagger J^\lambda|\Lambda\rangle \rightarrow \langle p|\mathcal{J}_\lambda{}^\dagger|\Lambda\rangle\langle\pi^-|J^\lambda|0\rangle$$

is approximately valid, apart from an absolute scale factor. There is at present no inconsistency in assuming that this reduction factor is no more than the momentum dependence of the vertex function $\langle p|\mathcal{J}_\Lambda{}^\dagger|\Lambda\rangle$ and the π^- decay matrix element $\langle\pi^-|J^\lambda|0\rangle$. If this method of resolution (which, incidentally, illustrates the connection of this process to pion decay in Born approximation) were accepted as a reasonable approximation to

[17] H. L. Anderson, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, Inc., New York, 1956); H. L. Anderson and W. C. Davidon, Nuovo cimento **5**, 1238 (1957).

[18] At higher energies where α_r become complex, Δ_r are still real but smaller in absolute magnitude (for real α_r, $\Delta_\gamma=\alpha_\gamma$).

[19] R. H. Dalitz, Phys. Rev. **112**, 605 (1958).

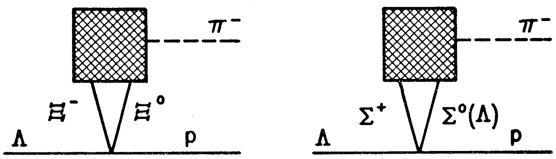

FIG. 9. Two diagrams contributing only to the charged decay mode.

the correct result, one could obtain information concerning the chiral coupling of the physical baryons in the strangeness-violating vertex.

Let us recall in this connection the structure of the vertex function in β decay, which involves a very small momentum transfer. We postulated that the interaction current J^λ is a (charged) chiral current. The vertex function $\langle p|J^\lambda|n\rangle$, which is the one-nucleon matrix element of this operator, is nearly equal to the Born approximation result, both in structure and in magnitude, since the ratio of the vector to axial vector covariants in the physical transition amplitude is nearly equal to unity. Also the absolute magnitude of the coefficients in the vertex function is equal to the "bare" coupling constant occurring in the Lagrangian operator as determined from the muon decay (and the hypothesis of a universal Fermi interaction). At the higher momentum transfer associated with π decay, there is a damping of the axial vector part of J^λ.

If the factorization of the matrix element for Λ decay is valid, one can easily see that only the axial vector part of J^λ contributes and that the large value of the asymmetry parameter α_- essentially implies chiral coupling in the vertex. There is also some damping in this decay process. That the absolute scale factor is determined by a momentum-dependent damping of the vertex function is consistent also with the apparent reduction of lepton modes.

If the above were a true interpretation of the experimental results, one would then have a consistent picture of the various transition amplitudes in the decay processes all reflecting the chiral structure of the interaction Lagrangian with the absolute scale factors dependent on the momentum transfer at the vertex. Whether one could arrange the structure of the strong interactions so as to ensure this unified point of view is still an open question.

ACKNOWLEDGMENTS

We wish to acknowledge conversations with Dr. C. J. Goebel, Mr. B. Sakita, Dr. S. B. Treiman, Dr. W. B. Teutsch, and Dr. S. Weinberg on the subject of this paper.

APPENDIX. SOLUTION OF THE SINGULAR INTEGRAL EQUATIONS

Here we wish to indicate the method used to solve the coupled singular integral equations derived in the text. The equations are [see Eqs. (29)]

$$F_1(\omega)=(m-\omega)f_3+\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left\{\frac{\mathrm{Re}[\varphi_3^*(\omega')F_1(\omega')]}{\omega'-\omega-i\epsilon}+\frac{\mathrm{Re}[\varphi_{31}^*(\omega')F_2(\omega')]}{\omega'+\omega}\right\},$$
$$F_2(\omega)=(m+\omega)f_3+\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left\{\frac{\mathrm{Re}[\varphi_3^*(\omega')F_1(\omega')]}{\omega'+\omega}+\frac{\mathrm{Re}[\varphi_{31}^*(\omega')F_2(\omega')]}{\omega'-\omega-i\epsilon}\right\}. \quad \text{(A-1)}$$

The method adopted is originally due to Muskhelishvili[16] and consists of introducing a complex function of which F_1 and F_2 are boundary values and then determining this complex function in terms of its singularities.

Accordingly, let us introduce the complex functions $H_1(z)$ and $H_2(z)$ of a complex variable z, such that when z is along the real axis, they coincide with $F_1(\omega)$ and $F_2(\omega)$:

$$H_1(z)=(m-z)f_3+\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left\{\frac{\mathrm{Re}[\varphi_3^*(\omega')F_1(\omega')]}{\omega'-z}+\frac{\mathrm{Re}[\varphi_{31}^*(\omega')F_2(\omega')]}{\omega'+z}\right\},$$
$$H_2(z)=(m+z)f_3+\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left\{\frac{\mathrm{Re}[\varphi_3^*(\omega')F_1(\omega')]}{\omega'+z}+\frac{\mathrm{Re}[\varphi_{31}^*(\omega')F_2(\omega')]}{\omega'-z}\right\}. \quad \text{(A-2)}$$

One immediately verifies the relations

$$H_2(z)=H_1(-z),\quad H_1^*(z)=H_1(z^*),\quad H_2^*(z)=H_2(z^*),$$
$$F_1(\omega)=\lim_{\epsilon\to+0}H_1(\omega+i\epsilon),\quad (\omega\geq m+\mu),$$
$$F_2(\omega)=\lim_{\epsilon\to+0}H_2(\omega+i\epsilon),\quad (\omega\geq m+\mu). \quad \text{(A-3)}$$

From the defining equations for H_1 and H_2, for $\omega\geq m+\mu$, the jump across the real axis can be found:

$$H_1(\omega+i\epsilon)-H_1(\omega-i\epsilon)=2i\,\mathrm{Re}[\varphi_3^*(\omega)H_1(\omega+i\epsilon)].$$

Taking the complex conjugate of both sides and using the result $H_1^*(z)=H_1(z^*)$, we see that

$$H_1(\omega-i\epsilon)\{1+i\varphi_3(\omega)\}=H_1(\omega+i\epsilon)\{1-i\varphi_3^*(\omega)\},$$

or

$$\frac{H_1(\omega+i\epsilon)}{H_1(\omega-i\epsilon)}=\exp\{2i\Delta_3(\omega)\}, \quad \text{(A-4a)}$$

where

$$\Delta_3(\omega)=\frac{1}{2i}\ln\left(\frac{1+i\varphi_3(\omega)}{1-i\varphi_3^*(\omega)}\right) \quad \text{(A-4b)}$$

is equal to $\alpha_3(\omega)$ when $\alpha_3(\omega)$ is real; when $\alpha_3(\omega)$ is complex, $\Delta_3(\omega)$ continues to be real. Similarly

$$\frac{H_1(-\omega+i\epsilon)}{H_1(-\omega-i\epsilon)}=\exp\{-2i\Delta_{31}(\omega)\},$$

where

$$\Delta_{31}(\omega)=\frac{1}{2i}\ln\left(\frac{1+i\varphi_{31}(\omega)}{1-i\varphi_{31}^*(\omega)}\right).$$

If we now introduce the complex function

$$K(z)=\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega'\left[\frac{\Delta_3(\omega')}{\omega'-z}+\frac{\Delta_{31}(\omega')}{\omega'+z}\right], \quad \text{(A-5a)}$$

with jumps

$$\begin{aligned} K(\omega+i\epsilon)-K(\omega-i\epsilon)&=2i\Delta_3(\omega),\\ K(-\omega+i\epsilon)-K(-\omega-i\epsilon)&=-2i\Delta_{31}(\omega), \end{aligned} \quad \text{(A-5b)}$$

across the real axis for $m+\mu<\omega<\infty$, then

$$h(z)=H_1(z)\exp[-K(z)] \quad \text{(A.6)}$$

is continuous across the real axis:

$$h(\pm\omega+i\epsilon)=h(\pm\omega-i\epsilon), \quad (m+\mu<\omega<\infty);$$

further, $H_1(z)$ and $K(z)$ are analytic everywhere in the complex plane cut along the real axis from $-\infty$ to $-(m+\mu)$ and from $(m+\mu)$ to $+\infty$; now the reality condition on $H_1(z)$ implies

$$h^*(z)=h(z^*),$$

enabling us to conclude that $h(z)$ is analytic everywhere in the complex plane, except possibly at infinity.

The expression for $K(z)$ will be meaningless unless the defining integrals converge and this requires that $\Delta_3(\omega)$ and $\Delta_{31}(\omega)$ decrease sufficiently fast at infinity; we assume this to be the case (at least after a suitable cutoff of the high-momentum contributions).

Let us now investigate the behavior of $h(z)$ at infinity. If the phase functions $\Delta_3(\omega)$, $\Delta_{31}(\omega)$ vanish sufficiently fast, for $z\to\infty$, $K(z)\to 0$ and $h(z)\to H_1(z)\to -f_3z$ and consequently $h(z)$ has a simple pole at infinity. However, since $h(z)$ possesses no essential singularities in the complex plane, the number of poles and zeros must be equal and hence $h(z)$ has a zero, say at $z=\theta$. If we now introduce

$$\psi(z)=\frac{1}{(-f_3)}\frac{h(z)}{z-\theta}, \quad \text{(A-7)}$$

then $\psi(z)$ is analytic everywhere, including the point at infinity, and is therefore a constant. For $z\to\infty$, $\psi(z)\to 1$ and hence,

$$\psi(z)\equiv 1, \quad H_1(z)=f_3(\theta-z)\exp[K(z)]. \quad \text{(A-8)}$$

It follows immediately that

$$\begin{aligned} F_1(\omega)&=f_3(\theta-\omega)\exp[K(\omega)+i\Delta_3(\omega)],\\ F_2(\omega)&=f_3(\theta+\omega)\exp[K(-\omega)+i\Delta_{31}(\omega)]. \end{aligned} \quad \text{(A-9)}$$

The zero of $h(z)$ is given by

$$\theta=m+\frac{1}{\pi}\int_{m+\mu}^{\infty}d\omega\{\sin\Delta_{31}(\omega)\exp[K(-\omega)] -\sin\Delta_3(\omega)\exp[K(\omega)]\}. \quad \text{(A-10)}$$

This completes the solution of the integral equation.[20]

It is gratifying to note that the phase factor of the amplitude is given by $\Delta(\omega)$. Consequently, below the threshold for real processes, the phase coincides with $\alpha(\omega)$ and the phases of the amplitudes for weak processes (which are invariant under time reversal) are essentially determined by the strong interactions—a well-known result.

[20] After our completion of this work, we noted a paper by R. Omnes [Nuovo cimento 8, 316 (1958)] in which he has independently arrived at the same technique of solution for similar integral equations.