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The Study of Spurious Scattering in Nuclear Emulsions and the Effect of Higher Order Differences in Scattering Measurements (*).

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Summary. — By considering the higher order differences in the scattering measurement, a new approach of reducing the spurious scattering in nuclear emulsions is discussed. The possible effect of turbulence during the developing stage is discussed in brief.

1. – Introduction.

The effect of spurious scattering in nuclear emulsions was first pointed out by BISWAS *et al.* [1] and since then there have been several methods used by different laboratories in trying to reduce the amount of spurious scattering. With the advance of the high-energy machines it has become quite necessary to reduce this spurious scattering to a value as low as possible so as to increase the signal to noise ratio without seriously reducing the statistical accuracy of the measurement. The methods to reduce the spurious scattering have mostly considered the effect of the development procedure on the presence of such a « distortion ». There appears to be some correlation between these two but at the same time there does not exist any clear understanding

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(1) S. BISWAS, B. PETERS and RAMA: *Proc. Ind. Acad. Sci.*, **41** A, 154 (1955).

of this phenomenon. The general features of spurious scattering have been summarized by BISWAS *et al.* [2] as follows:

1) Spurious scattering is present in all emulsions. It is not possible to eliminate spurious scattering for individual tracks from a knowledge of the mean value of spurious scattering.

2) The mean value of spurious scattering differs from stack to stack. The largest and smallest mean values observed so far differ by a factor slightly smaller than two.

3) Spurious scattering varies with cell length, increasing with increase in cell length.

We present here another possible method of reducing the so-called spurious scattering by taking an approach quite similar to that employed previously for the low energy particles *viz.* the removal of distortion in nuclear emulsions by going to higher order differences in the scattering measurements.

2. – Experimental procedure.

We have used part of the data obtained during the analysis given by JUDEK [3] where a correlation between the development temperature and the amount of spurious scattering present in the nuclear emulsions is demonstrated. Various batches of K-5 and G-5 emulsions were exposed to the 6.2 GeV proton beam from the Berkeley Bevatron and each batch was developed differently. The details of the development procedures used are given in ref. [3].

In order to consider the effect of higher order differences only, we have chosen the data for the two lowest development temperatures *viz.* 5 °C and 10 °C, where the spurious scattering was considered to be the least. The higher order differences for various tracks were calculated for four different cell sizes 500 μm, 1000 μm, 2000 μm, and 4000 μm. The 7070 computer was used for these simple but tedious calculations. Overlapping readings have not been used to obtain the higher cell length scattering parameter $\bar{D}$, hence the number of independent readings decreases rapidly as we go to higher cell lengths.

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[2] S. BISWAS, N. DURGA PRASAD and S. MITRA: *Proc. Ind. Acad. Sci.*, **46** A, 167 (1957).

[3] B. JUDEK: *Nuovo Cimento*, **16**, 834 (1960).

3. – Results and discussion.

The observed values of the mean sagitta of scattering in various orders n, and for a particular cell size t is denoted by $\bar{D}_n(t)$. A list of these values for the different batches of emulsion is tabulated in Table I. We have not used a $4\bar{D}$ cut-off procedure in the present analysis and as a result, the observed values appear slightly larger than those obtained in ref. ([3]). We do not consider the effect to be very serious for the purpose of our present calculations.

The theoretical values of sagittae of multiple scattering of 6.2 GeV protons are denoted by $\Delta_n(t)$ and for $n=2$, these values have been tabulated in Table II. The effect of the slight increase in the scattering constant with the cell length is also included in obtaining these values ([4]).

We now consider the variation of $\Delta_n(t)$ as a function of n. Assuming that multiple scattering sagittae obey a correlated multivariate gaussian distribution, we have calculated the ratio $\Delta_n(t)/\Delta_2(t)$ for different values of n. The details of these calculations are given in Appendix I. The results are summarized in Table III. Using the values of $\bar{D}_n(t)$ and $\Delta_n(t)$ for various n, we obtain the quantity $\varepsilon_n(t)$ which has been called the « total noise » defined as follows:

$$\varepsilon_n(t) = \sqrt{\bar{D}_n^2(t)) - \Delta_n^2(t)} \,. \tag{1}$$

It should be noted that this « total noise » consists of the reading, grain and the stage noise together with the so called spurious scattering.

The values of $\varepsilon_n(t)$ so obtained are then used to calculate the quantity $\varepsilon_2'(n, t)$ which we call the « reduced total noise » and define as follows:

$$\varepsilon'_{2(n,t)} = \frac{\varepsilon_n(t)}{N(n)} \,, \tag{2}$$

where $N(n)$ gives the reduction ratio of the noise in the n-th order to that in the 2-nd order. $N(n)$ has been theoretically calculated for different values of n, assuming that the distribution of grains around the particle trajectory, which then gives rise to the non-Coulomb contribution to the observed values of mean sagitta of scattering, is purely random. The details of these calculations are given in Appendix II. We have summarized these results in Table IV which gives the value of $N(n)$ for various values of n. It is obvious that $N(n)$

([4]) K. GOTTSTEIN, M. G. K. MENON, J. H. MULVEY, C. O'CEALLAIGH and O. ROCHAT: *Phil. Mag.*, **42**, 708 (1951).

TABLE I. – *Observed values of the mean sagitta of scattering, in various orders, for* 6.2 GeV *protons* ($\bar{D}_n(t)$).

Batch	Type of emulsions and method of development as given in ref. [3]	Observed mean sagittae $\bar{D}_n$ (μ) / Cell length	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	$\bar{D}_6$	$\bar{D}_8$
I	G-5 Z 1401 5 °C Isothermal (*a*)	500 μm	0.16±0.01	0.24±0.02	0.42±0.03	1.46±0.09	5.32± 0.37
		1000 μm	0.35±0.02	0.47±0.03	0.76±0.05	2.58±0.17	9.16± 0.64
		2000 μm	0.91±0.08	1.12±0.11	1.90±0.18	6.41±0.67	23.59± 2.62
		4000 μm	2.90±0.41	3.76±0.57	6.17±0.99	19.40±3.60	65.90±15.10
II	G-5 Z 1401 5 °C Isothermal (*b*)	500 μm	0.13±0.01	0.22±0.01	0.40±0.02	1.47±0.08	5.49± 0.28
		1000 μm	0.27±0.02	0.36±0.02	0.60±0.04	1.98±0.13	6.84± 0.46
		2000 μm	0.69±0.06	0.92±0.09	1.56±0.15	5.29±0.55	18.37± 2.04
		4000 μm	1.95±0.30	2.57±0.42	4.13±0.71	13.73±2.69	44.43± 8.39
III	G-5 Z 1401 10 °C Isothermal (*a*)	500 μm	0.17±0.01	0.28±0.01	0.50±0.02	1.74±0.09	6.49± 0.35
		1000 μm	0.31±0.02	0.44±0.03	0.76±0.05	2.63±0.17	9.77± 0.64
		2000 μm	0.82±0.07	1.12±0.10	1.90±0.18	6.20±0.60	22.63± 2.37
		4000 μm	2.32±0.29	3.32±0.44	5.58±0.80	16.83±2.71	55.21±10.85
IV	G-5 Z 1410 10 °C Isothermal (*b*)	500 μm	0.13±0.01	0.21±0.01	0.37±0.02	1.31±0.08	4.89± 0.31
		1000 μm	0.28±0.02	0.41±0.03	0.71±0.05	2.57±0.18	9.68±0.70
		2000 μm	0.70±0.07	0.85±0.09	1.39±0.14	4.41±0.48	15.33± 1.76
		4000 μm	2.19±0.33	2.37±0.38	4.01±0.67	14.30±2.75	57.47±13.15
V	G-5 Z 1401 10 °C Temperature cycle	500 μm	0.16±0.01	0.23±0.01	0.40±0.02	1.42±0.06	5.20± 0.21
		1000 μm	0.35±0.02	0.48±0.02	0.82±0.04	2.64±0.13	9.53± 0.47
		2000 μm	0.91±0.06	1.20±0.08	2.00±0.14	6.80±0.50	24.53± 1.91
		4000 μm	2.54±0.24	3.23±0.32	5.45±0.57	18.27±2.18	62.19± 8.80

TABLE II. – *Theoretical values of the Coulomb multiple scattering for* 6.2 GeV *protons* $\Delta_2(t)$.

Cell length	500 μm	1 000 μm	2 000 μm	4 000 μm
$\Delta_2(t)$ μm	0.070	0.207	0.605	1.76

TABLE III. – *The ratio of the higher order differences of the Coulomb multiple scattering sagittae to those in the second order.*

n	2	3	4	5	6	7	8
$\Delta_n(t)/\Delta_2(t)$	1.0	1.223	2.00	3.53	6.47	12.11	23.1

TABLE IV. – *The ratio of higher order differences of the noise readings to those in the second order* $N(n)$.

n	2	3	4	5	6	7	8
$N(n)$	1.00	1.828	3.416	6.47	12.41	23.9	46.9

is assumed to be independent of the cell size and hence not strictly applicable to our problem due to the presence of stage noise. It may be pointed out here that BISWAS *et al.* ([1,2]) and JUDEK ([3]) have assumed that the stage noise is negligibly small for the MS-2 type Koristka miscrocope. They have also not considered the variation of this stage noise with the cell size. Later work of GUSS ([5]), using a R-4 type Koristka microscope, has shown that the stage noise indeed increases with the cell size. Recent results presented by the Brussels group ([6]) indicate that even for a MS-2 Koristka microscope, the stage noise increases with cell size and changes from 0.022 μm to 0.146 μm as we increase the cell length from 500 μm to 4 000 μm. If this effect is taken into account then the results of ref. ([1-3]) will be slightly modified. We will discuss the effect of this observation in our present calculations at a later stage.

In order to note the effect of the higher order differences in reducing the noise level, we have to compare the values $\varepsilon_2'(n, t)$ and $\varepsilon_2(t)$; the latter being the « noise » most commonly measured by different workers while the former is that obtained by the consideration of higher order differences. This com-

([5]) D. E. GUSS: *Ph. D. Thesis*, Washington University, St. Louis (1961).
([6]) Informal meeting held in Lausanne (1961).

parison is made for different batches of emulsions and results are shown in Fig. 1(*a*) to 5(*a*). We have also shown, for comparison. the variation of the true multiple scattering $\Delta_2(t)$ with the cell length. The error on each point is obtained by taking an error of $100/\sqrt{m(n,t)}\,\%$ on $\bar{D}_n(t)$ where $m(n,t)$ denotes the number of independent readings for a particular cell length and for the n-th order differences. Figures 1(*b*) to (5*b*) also demonstrate this effect of

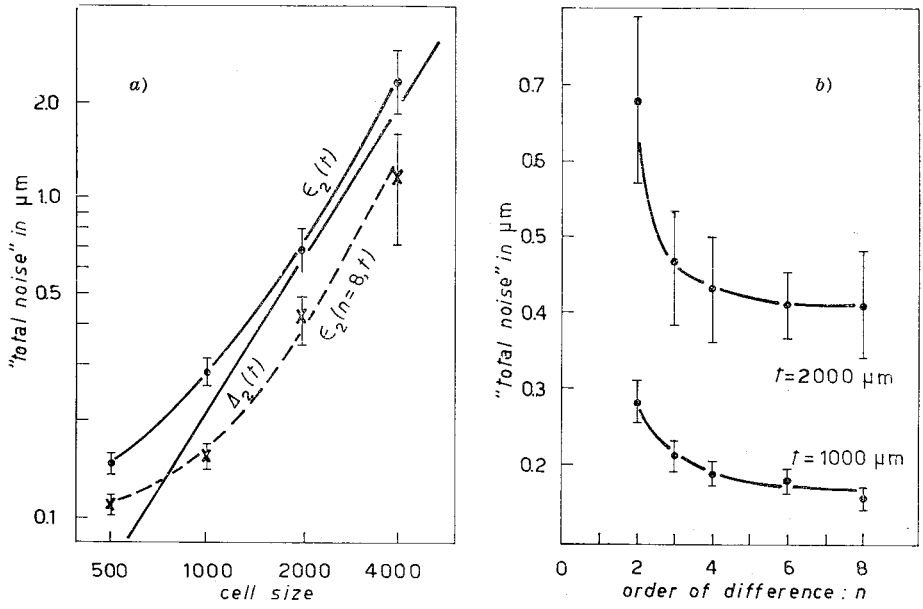

Fig. 1. – *a*) The behaviour of the « total noise » as a function of cell length for batch I emulsions. The curved solid line represents the values of the total noise as obtained from consideration of second differences only, while the broken line indicates the variation of the « reduced total noise » as obtained from higher order differences ($n=8$). The solid straight line represents the variation of the multiple Coulomb scattering $\Delta_2(t)$ with cell length. *b*) The variation of the « total noise » with various orders of differences from which it has been determined. Curves for only two cell sizes have been shown for batch I.

reduction of the total noise as a function of the different orders for a given cell length. It can be seen from these figures that the higher order differences effectively reduce the noise level as one goes from second to the third difference but gradually attains a fairly constant value beyond the fourth or the fifth difference. However, there still remains the major difficulty of explaining the dependance of the « total noise » on the cell length as is clearly seen from the fact that there are different curves for different cell lengths. (Except in one case where the two curves are very close to each other). Since

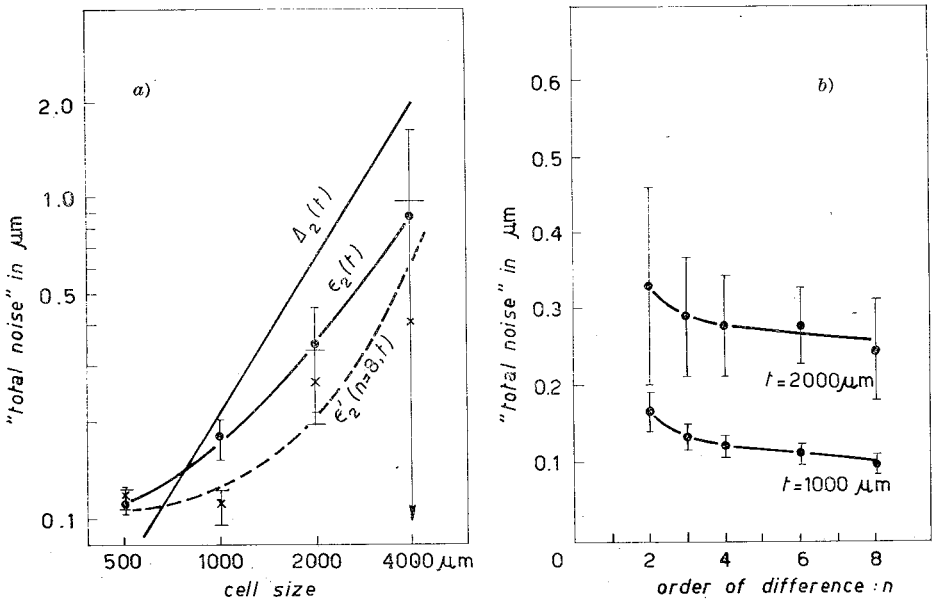

Fig. 2. – *a*) The behaviour of the « total noise » as a function of cell length for batch II emulsions. *b*) The variation of the « total noise » with various orders of differences from which it has been determined. Curves for only two cell sizes have been shown for batch II.

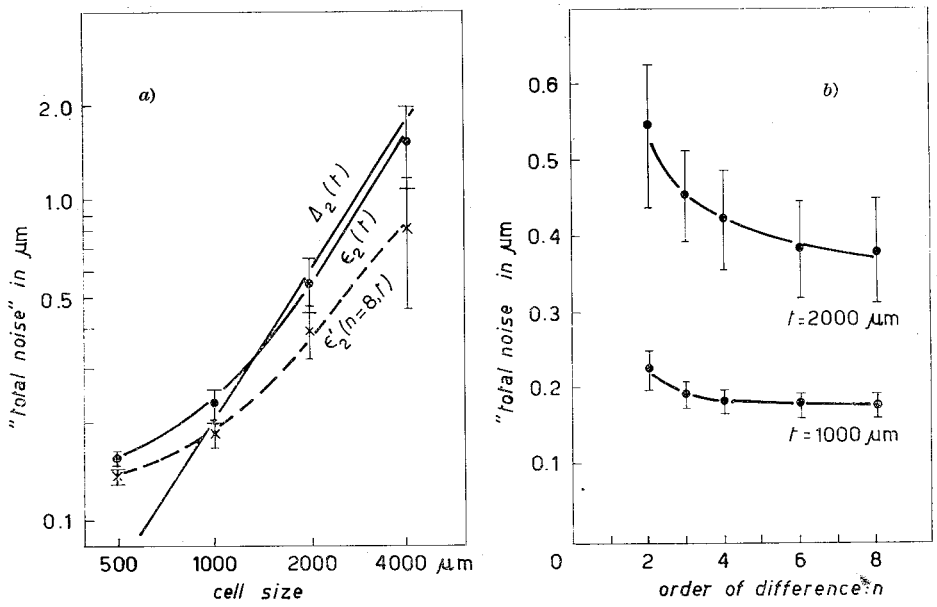

Fig. 3. – *a*) The behaviour of the « total noise » as a function of cell length for batch III emulsions. *b*) The variation of the « total noise » with various orders of differences from which it has been determined. Curves for only two cell sizes have been shown for batch III.

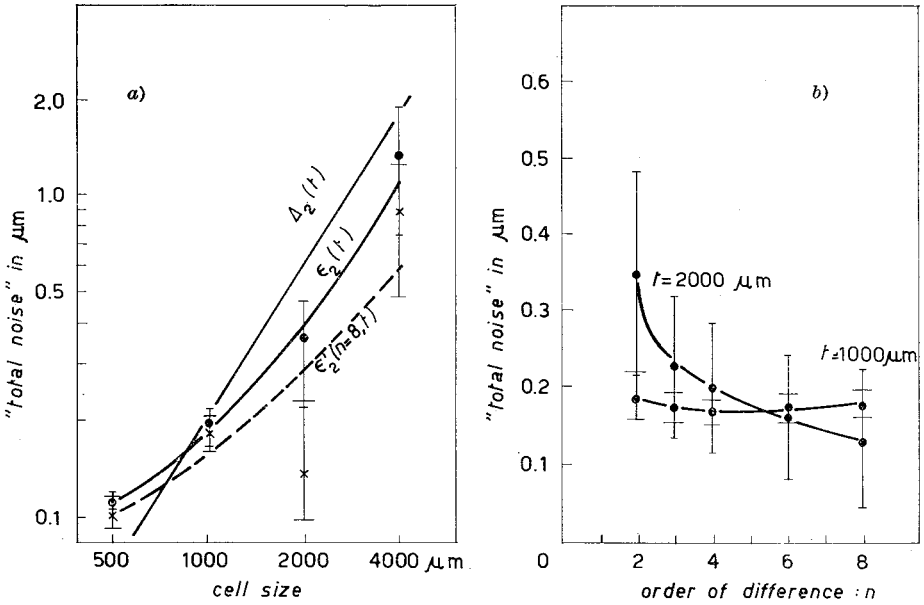

Fig. 4. – *a*) The behaviour of the « total noise » as a function of cell length for batch IV emulsions. *b*) The variation of the « total noise » with various orders of differences from which it has been determined. Curves for only two cell sizes have been shown for batch IV.

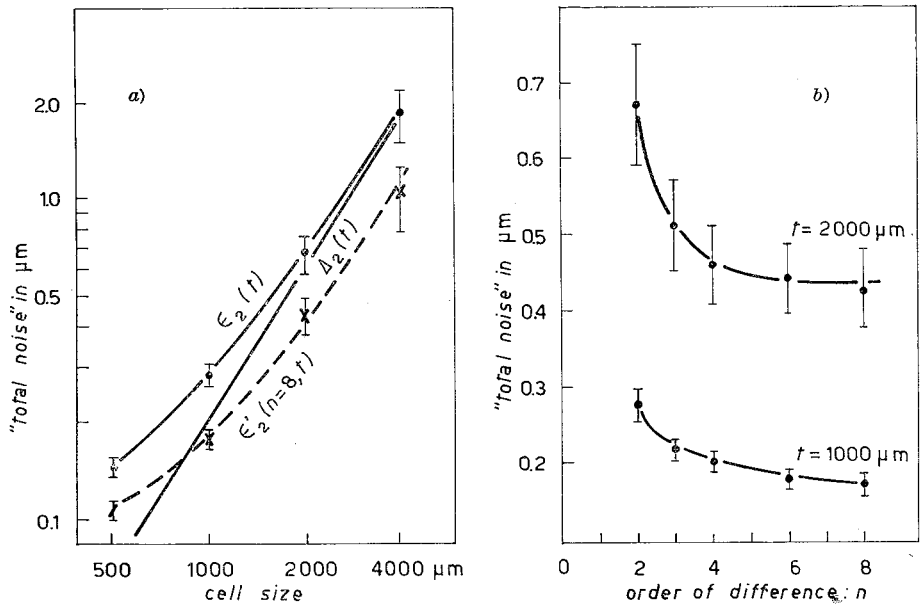

Fig. 5. – *a*) The behaviour of the « total noise » as a function of cell length for batch V emulsions. *b*) The variation of the « total noise » with various orders of differences from which it has been determined. Curves for only two cell sizes have been shown for batch V.

the reading and grain noise cannot depend on the cell length, the dependance must come from the stage noise and the so-called spurious scattering.

In order to consider only the variation of the spurious scattering with the cell size, we have assumed the following relation defining the spurious scattering $\bar{D}'_{ss}(n, t)$

$$\bar{D}'_{ss}(n = 8, t) = \sqrt{\varepsilon_2'^2(n = 8, t) - \bar{D}_\varepsilon^2 - \bar{D}_\gamma^2}\,, \tag{3}$$

where we have used the standard definition for $\bar{D}_\varepsilon$ and $\bar{D}_\gamma$ as given in ref. (6). They are as follows:

$\bar{D}_\varepsilon$ = mean absolute value of second differences of grain, reading an irreproducible noise; independent of cell length,

and

$\bar{D}_\gamma$ = mean absolute value of second differences of the reproducible stage noise.

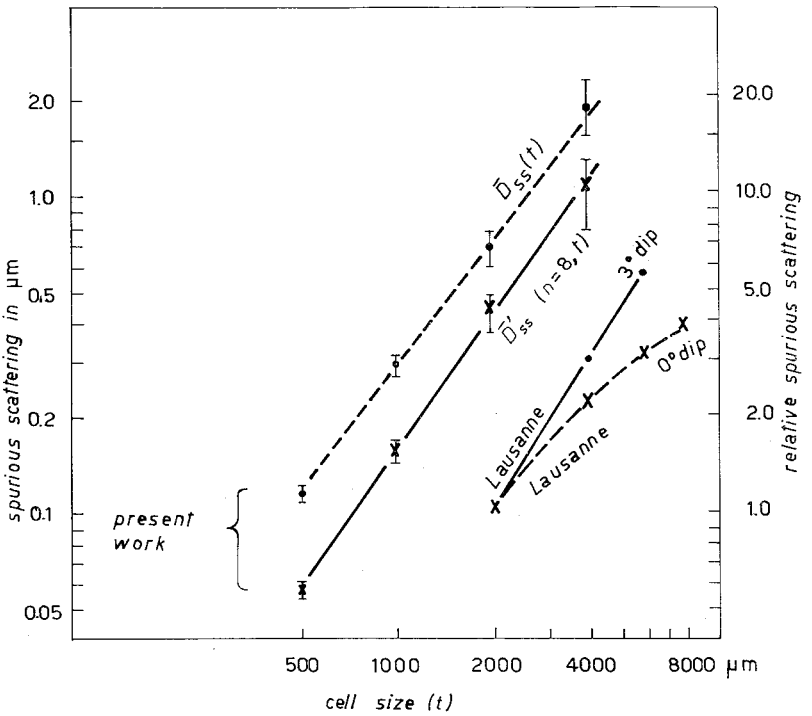

Fig. 6. – The mean value of spurious scattering as a function of cell size for the emulsion batch V. The broken line is drawn for spurious scattering as derived from the second order differences $[\bar{D}_{ss}(t)]$, while the solid line represents the « reduced spurious scattering » $\bar{D}'_{ss}(n=8, t)$, as obtained from the eighth order differences. The results of the Lausanne group are also shown for two different dip angles. (Their values are given relative to the value for the cell size of 2 mm).

We have used the mean value of $\bar{D}_\varepsilon$ for G-5 emulsions given by JUDEK [3] ad $\bar{D}_\varepsilon = 0.087$ μm. The values of $\bar{D}_\gamma$ for a Koristka MS-2 microscope as given by the Brussels group [6] are used since such measurements were not made by JUDEK [3]. The values of $\bar{D}'_{ss}$ $(n = 8, t)$ as obtained from eq. (3) are plotted in Fig. 6 for a single batch for which we have maximum number of readings. For comparison, we have also shown, by a broken line, the values of $\bar{D}_{ss}(t)$ which would have been observed if we had not considered the effect of higher order differences and defined the spurious scattering as strictly given in ref. [6]. (The comparison is still not exact since we have not considered the effect of a $4\bar{D}$ cut-off.) This value of $\bar{D}_{ss}(t)$ is given by the relation

$$\bar{D}_{ss}(t) = \sqrt{\bar{D}_2^2(t) - \bar{\Delta}_2^2(t) - \bar{D}_\varepsilon^2 - \bar{D}_\gamma^2}. \tag{4}$$

We observe that $\bar{D}'_{ss}$ $(n = 8, t)$ is still a function of the cell size, varying roughly at $t^{1\cdot4}$ while $\bar{D}_{ss}(t)$ varies as $t^{1\cdot28}$ but the magnitude of the spurious scattering is definitely reduced by going to the higher order differences. We have also shown the variation of spurious scattering with cell length as measured by the Lausanne group [6] for tracks of different dip angles. The results are given in terms of the spurious scattering for the basic cell length of 2 mm.

4. – Conclusion.

In conclusion we would like to state that the method of higher differences seems to help in reducing the level of spurious scattering considerably though it does not completely eliminate it. The cell size dependence of the « reduced total noise » cannot be explained purely by the variation of the stage noise since this contribution is not sufficiently large and hence we still have to invoke the hypothesis about the existence of the so-called spurious scattering and its variation with cell length.

We may think of the spurious scattering as being due to a turbulent displacement of the emulsion grains consequent on the chemical changes undergone by the emulsion in the process of development and fixing. If we analyse these turbulent displacements in terms of eddy sizes we may distinguish, in relation to the scattering cell length, three ranges: large, small and medium. For the large eddies, going to higher order will essentially eliminate the turbulent noise; for the small eddies, the turbulent noise behaves like any other noise in that the contribution to successive readings are uncorrelated. The relative contributions to the scattering in various orders due to the medium size eddies is more complicated and depends on the details of the turbulence spectrum in this range of eddy sizes. The residual noise $\varepsilon'_2(n, t)$ obtained from higher order differences contains contributions from the small and medium

size eddies. As the cell length is increased, less of turbulent displacements remain « large » eddies. The increase of the turbulent noise with an increase of cell size may now be interpreted to mean that the turbulent displacements are still appreciable at eddy sizes of the order of the largest cell lengths that we have used.

If this is indeed the mechanism, the following points need further investigation: 1) Does the noise $\varepsilon_2'(n, t)$ saturate with cell length? If there is a cut-off in the turbulence at some eddy size, such a saturation must eventually be obtained. 2) If the turbulent displacements cause spurious scattering, then, for the same emulsion batch, the noise $\varepsilon_2'(n, t)$ must be the same for tracks of different energies. But it may be different for different emulsion batches, and particularly so for emulsions of different compositions. These investigations are currently being planned.

* * *

We are greatly indebted to Dr. B. JUDEK for providing us with the scattering data and for many useful comments. We would also like to express our sincere thanks to Dr. S. ROSENDORFF for making some very useful suggestions in this work. We take pleasure in thanking Drs. M. V. K. APPA RAO, M. F. KAPLON and G. YEKUTIELI for many useful discussions. We deeply appreciate the help of Mr. SURESH THAKORE in the use of the 7070 computer. Miss JUNE CHRISTENSEN helped us in making a large number of calculations and we are grateful to her for her patient work.

APPENDIX I

The arithmetic average of higher order differences of multiple scattering sagittae.

W. T. SCOTT [7] has shown that, in the multiple scattering approximation, the values of n successive second differences x_r, $r=1, 2 \ldots n$ has a correlated multivariate Gaussian distribution of the form

$$P^{(2)}(x_1, \ldots x_r)\,\mathrm{d}x_1 \ldots \mathrm{d}x_r = \mathrm{Const}\,\mathrm{d}x_1 \ldots \mathrm{d}x_r \exp\left[\frac{-x_r B_{rs}^{(2)} x_s}{\alpha^2}\right],$$

(7) W. T. SCOTT: *Phys. Rev.*, **76**, 212 (1949).

with the $n \times n$ matrix $B^{(2)}_{r,s}$ given by

$$B^{(2)} = A^{(2)^{-1}}; \qquad A^{(2)}_{rs} = 4\delta_{rs} + \delta_{r,s+1} + \delta_{r+1,s} \quad i.e.,$$

$$A^{(2)} = \begin{vmatrix} 4 & 1 & 0 & 0 & \ldots \\ 1 & 4 & 1 & 0 & \ldots \\ 0 & 1 & 4 & 1 & \ldots \\ 0 & 0 & 1 & 4 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{vmatrix}.$$

We can show that similarly for the $(n - m + 2)$ successive m-th differences we have a similar probability distribution of the form $\exp[-x_r B^{(m,n)}_{r,s} x_s/\alpha^2]$ with a suitable $B^{(m,n)}_{r,s}$.

To caculate $B^{(m,n)}_{r,s}$ we define the $n' \times n' - 1$ real matrix $c_{r,s}(n')$ with elements $\delta_{r,s} - \delta_{r,s-1}$ *i.e.*

$$c(n') = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & \ldots \\ -1 & 1 & 0 & 0 & 0 & \ldots \\ 0 & -1 & 1 & 0 & 0 & \ldots \\ 0 & 0 & -1 & 1 & 0 & \ldots \\ 0 & 0 & 0 & -1 & 1 & \ldots \end{vmatrix},$$

$$c^T(n') = \begin{vmatrix} 1 & -1 & 0 & 0 & \ldots \\ 0 & 1 & -1 & 0 & \ldots \\ 0 & 0 & 1 & -1 & \ldots \\ 0 & 0 & 0 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{vmatrix}.$$

Then it follows that $B^{(m,n)} = A^{(mn)^{-1}}$ with

$$A^{(2,n)} \equiv A^{(2)}; \qquad A^{(3,n)} = c^T(n-1) A^{(2,n)} c(n-1);$$

$$A^{(4,n)} = c^T(n-2) A^{(3,n)} c(n-2); \qquad A^{(5,n)} = c^T(n-3) A^{(4,n)} c(n-3).$$

More generally

$$A^{(m,n)} = c^T(n+2-m) A^{(m-1,n)} c(n+2-m)$$

$$= c^T(n+2-m) c^T(n+3-m) \ldots c^T(n-1) A^{(2,n)} c(n-1) \ldots c(n+2-m).$$

Calculating directly in this manner we get

$$A^{(2,n)}_{r,s} = 4\delta_{rs} + \delta_{r,s+1} + \delta_{r+1,s},$$

$$A^{(3,n)}_{r,s} = 6\delta_{rs} - 2\ (\delta_{r,s+1} + \delta_{r+1,s}) - (\delta_{r,s+2} + \delta_{r+2,s}),$$

$$A^{(4,n)}_{r,s} = 16\delta_{rs} - 9\ (\delta_{r,s+1} + \delta_{r+1,s}) + (\delta_{r,1+3} + \delta_{r+3,s}),$$

$$A^{(5,n)}_{r,s} = 50\delta_{rs} - 34(\delta_{r,s+1} + \delta_{r+1,s}) + 8(\delta_{r,s+2} + \delta_{r+2,s}) + 2(\delta_{r,s+3} + \delta_{r+3,s}) - (\delta_{r,s+4} + \delta_{r+4,s}),$$

etc.
These matrices are written out below explicitly.

$$A^{(2,n)} = \begin{vmatrix} 4 & 1 & 0 & 0 & \dots \\ 1 & 4 & 1 & 0 & \dots \\ 0 & 1 & 4 & 1 & \dots \\ 0 & 0 & 1 & 4 & \dots \\ 0 & 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix},$$

$$A^{(3,n)} = \begin{vmatrix} 6 & -2 & -1 & 0 & 0 & \dots \\ -2 & 6 & -2 & -1 & 0 & \dots \\ -1 & -2 & 6 & -2 & -1 & \dots \\ 0 & -1 & -2 & 6 & -2 & \dots \\ 0 & 0 & -1 & -2 & 6 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix},$$

$$A^{(4,n)} = \begin{vmatrix} 16 & -9 & 0 & 1 & 0 & \dots \\ -9 & 16 & -9 & 0 & 1 & \dots \\ 0 & -9 & 16 & -9 & 0 & \dots \\ 1 & 0 & -9 & 16 & -9 & \dots \\ 0 & 1 & 0 & -9 & 16 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix},$$

$$A^{(5,n)} = \begin{vmatrix} 50 & -34 & 8 & 2 & -1 & 0 & \dots \\ -34 & 50 & -34 & 8 & 2 & -1 & \dots \\ 8 & -34 & 50 & -34 & 8 & 2 & \dots \\ 2 & 8 & -34 & 50 & -34 & 8 & \dots \\ -1 & 2 & 8 & -34 & 50 & -34 & \dots \\ 0 & -1 & 2 & 8 & -34 & 50 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix},$$

$$A^{(6,n)} = \begin{vmatrix} 168 & -126 & 48 & -3 & -4 & 1 & \dots \\ -126 & 168 & -126 & 48 & -3 & -4 & \dots \\ 48 & -126 & 168 & -126 & 48 & -3 & \dots \\ -3 & 48 & -126 & 168 & -126 & 48 & \dots \\ -4 & -3 & 48 & -126 & 168 & -126 & \dots \\ 1 & -4 & -3 & 48 & -126 & 168 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix},$$

$$A^{(7,n)} = \begin{vmatrix} 588 & -468 & 225 & -50 & -6 & \dots \\ -468 & 588 & -468 & 225 & -50 & \dots \\ 225 & \dots & \dots & \dots & \dots & \dots \\ -50 & \dots & \dots & \dots & \dots & \dots \\ -6 & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix},$$

$$A^{(8,n)} = \begin{vmatrix} 2112 & -1749 & 968 & -319 & 32 & 19 & \dots \\ -1749 & 2112 & -1749 & 968 & -319 & 32 & \dots \\ 968 & \dots & \dots & \dots & \dots & \dots & \dots \\ -319 & \dots & \dots & \dots & \dots & \dots & \dots \\ 32 & \dots & \dots & \dots & \dots & \dots & \dots \\ 19 & \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix}.$$

It can now be seen, on choosing $n = m-1$ thet if the arithmetic mean values of the m-th differences are denoted by Δ_m then

$$\frac{\Delta_m}{\Delta_2} = \frac{1}{2}\sqrt{A_{11}^{(m,n)}},$$

where $A_{11}^{(m,n)}$ are the diagonal elements of the matrix $A_{r,s}^{(m,n)}$ (recalling that $A_{11}^{(2,n)}=4$). It then follows that

$$\frac{\Delta_3}{\Delta_2} = \sqrt{\frac{6}{4}}; \qquad \frac{\Delta_4}{\Delta_2} = \sqrt{\frac{16}{4}} \text{ etc.},$$

and the results of these computations are presented in Table III in the text.

APPENDIX II

The arithmetic average of higher order differences of noise readings.

We shall assume that successive primitive noise readings y_n are uncorrelated. The noise sagittae are given by the formula

$$\varepsilon_n^{(2)} = (y_n - 2y_{n+1} + y_{n+2}) .$$

In the Gaussian approximation, denoting by ε the arithmetic average of the noise sagittae, the values of m successive primitive noise readings in the uncorrelated Gaussian distribution:

$$P(y_1, \ldots y_m)\, \mathrm{d}y_1 \ldots \mathrm{d}y_m = \text{Const} \exp\left[-\frac{1}{6\pi\varepsilon^2} \sum_{r=1}^{m} y_r^2\right] \mathrm{d}y_1 \ldots \mathrm{d}y_m .$$

While the y_n are uncorrelated this is no longer true of $\varepsilon_n^{(2)}$, or more generally of $\varepsilon_n^{(m)}$. Rather than write down the correlated distributions for $\varepsilon_n^{(m)}$ it is easier to calculate the arithmetic average $\varepsilon_{(m)} = \langle |\varepsilon_n^{(m)}| \rangle$ directly from the distribution for the y_m using the explicit formula

$$\varepsilon_n^{(m)} = \sum_{r=0}^{m} \frac{m!}{r!\,(m-r)!} (-1)^r y_{n+r} .$$

Since for a Gaussian distribution with zero mean, the arithmetic average is $\sqrt{2/\pi}$ times the root mean square, recalling that y_n are uncorrelated, it follows that

$$\varepsilon_{(m)} = \sqrt{\sum_{r=0}^{m} \left\{\frac{m!}{r!\,(m-r)!}\right\}^2} \langle |y_n| \rangle = \frac{\sqrt{(2m)!}}{(m)!} \cdot \langle |y_n| \rangle .$$

In particular, $\varepsilon_{(2)} = \varepsilon$ as was to be expected. We thus obtain the relation

$$\varepsilon_{(n)} = \frac{\varepsilon}{n!} \sqrt{\frac{(2n)!}{6}} = N(n)\varepsilon = N(n)\varepsilon_{(2)} ,$$

with

$$N(n) = \frac{1}{n!} \sqrt{\frac{(2n)!}{6}} .$$

The numerical values are given in Table IV in the text.

The Gaussian approximation used here may not necessarily be true for the primitive noise readings. A numerical study ([8]) of the deviation of the shape of the distribution curves in various orders suggests that the deviations are not expected to be significant in altering the ratio of the arithmetic averages in various orders.

([8]) E. C. G. SUDARSHAN: unpublished.

RIASSUNTO (*)

Tenendo in considerazione le differenze di ordine più elevato nella misura dello scattering, si discute un nuovo tentativo di ridurre lo scattering spurio nelle emulsioni nucleari. Si discute brevemente anche il possibile effetto della turbolenza durante lo sviluppo.

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