Algebraic models for weak interactions

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IT IS NOW CLEAR that the classification of particles into leptons, photons and hadrons; and of interactions into weak, electromagnetic and strong, is a useful device in particle physics. Since the hadrons participate in all the three kinds of interactions, hadron phenomena are richer and more complex; and, consequently, harder to analyze. To understand the weak or electromagnetic interactions of hadrons, it is essential that we have a reasonably good understanding of their strong interactions. If we have to look for pristine weak (or electromagnetic) interactions, we must examine the corresponding lepton phenomena.

As far as the purely leptonic weak interactions are concerned, we have known for sometime that it has a most simple structure: we simply pick up the positive chiral components of suitably selected pairs of leptons and form the only bilinear quantity that can be formed out of them. This quantity has a vectorial transformation property: it is a combination of a vector and of an axial vector. The leptonic weak interaction is obtained by simply contracting two such vectorial quantities. We thus get the chiral *V–A* coupling for muon beta decay:

$$\frac{G}{\sqrt{2}}\,(\bar{\mu}\gamma^{\lambda}\,(1+\gamma_5)\,\nu_{\mu})^{\dagger}\,(\bar{e}\gamma_{\lambda}\,(1+\gamma_5)\,\nu_e).$$

For the parallel case of nuclear beta decay one is tempted to write

$$\frac{G}{\sqrt{2}}\,(\bar{n}\gamma^{\lambda}\,(1+\gamma_5)\,p)^{\dagger}\,(\bar{e}\gamma_{\lambda}\,(1+\gamma_5)\,\nu_e)$$

with the same value of G. The approximate validity of this interaction structure in nuclear physics makes it all the more necessary to understand why the vector and axial vector strengths are not equal; and to search for a method of diciphering the phenomenology of hadron weak interactions. We shall confine our attention to semileptonic processes.

Like the Wizard of Oz said to the Cowardly Lion, there is one thing that hadrons have that the leptons do not have: membership in multiplets. And their strong interactions cannot be understood without taking account of such a multiplet structure. The multiplet structures themselves can be understood in terms of algebraic models. You will hear more about such models in the presentations by Professor Ne'eman and Professor Böhm.

I A MODEL FOR NUCLEAR BETA DECAY[1]

For our purposes we should look for a simple algebraic model involving the nucleons and in which both Fermi type (spin-independent) and Gamow–Teller type (spin-dependent) interactions can be included. The algebra must therefore involve both charge- and spin-changing operators. A simple such algebra is provided by $SU(4)$ with fifteen generators, divided into three sets of 3,3 and 9 which transform as σ_j, τ_α and $\sigma_j\tau_\alpha$ respectively. The nucleons together with the $I = J = 3/2$ nucleon resonances furnish a 20 dimensional symmetric tensor representation of this algebra.

To complete the picture we must also furnish a method of coupling mesons to this nucleonic multiplet. Since the treatment of spin in $SU(4)$ relies on the nonrelativistic limit we must look at the behavior of the meson couplings in the nonrelativistic limit. In such a limit the time component of a vector coupling, the space components of an axial vector coupling and the space–space components of a tensor coupling alone survive. For the relevant part of the meson–nucleon couplings we take the covariant interaction:

$$\tfrac{1}{2}g\bar{N}\,\{\tau_\alpha V_\alpha^\lambda\gamma_\lambda + (g'/g)\,\tau_\alpha V_\alpha^{\lambda\nu}\,\tfrac{1}{2}\sigma_{\lambda\nu} + (g'_\infty/g)\,\phi^{\lambda\nu}\,\tfrac{1}{2}\sigma_{\lambda\nu} + (f/g)\,\tau_\alpha A_\alpha^\lambda\gamma_\lambda\gamma_5 + (f'/g)\,\tau_\alpha A_\alpha^{\lambda\nu}\,\tfrac{1}{2}\sigma_{\lambda\nu}\gamma_5\}\,N.$$

In the nonrelativistic limit this reduces to the form

$$\tfrac{1}{2}gN^\dagger\,\{\tau_\alpha V_\alpha^0 + (g'/gm_v)\,\tau_\alpha\,(\boldsymbol{\sigma}\cdot\nabla\times\mathbf{V}_\alpha) + (g'_\infty/gm_\phi)\,(\boldsymbol{\sigma}\cdot\nabla\times\boldsymbol{\phi}) + (f/g)\,\tau_\alpha\,(\boldsymbol{\sigma}\cdot\mathbf{A}_\alpha) + (f'/gm_A)\,\tau_\alpha\,(\nabla A_\alpha^0 - \dot{\mathbf{A}}_\alpha)\}\,N.$$

We have to pick up a fifteen component meson wave function to be coupled to the $SU(4)$ nucleonic fields. Such a matrix is made up of the V_α^0 ($I=1$, $J=0$), $\dfrac{1}{m_\phi}\,\nabla\times\boldsymbol{\phi}$ ($I=0$, $J=1$) and a suitably normalized combination of $\mathbf{A}_\alpha$ and $\dfrac{1}{m_A}\,(\nabla A_\alpha^0 - \dot{\mathbf{A}}_\alpha)$. The simplest choice is to take a combination with equal weights. With these choices the $SU(4)$ invariant coupling is

$$(g'_\infty/g) = 1; \quad (f/g) = (f'/g) = \left(5/3\sqrt{2}\right).$$

Postulating that the weak interactions are induced by the direct coupling of the vector and axial vector meson fields with the leptons according to

$$(-G/g)\cdot(m_v^2 V_+^\lambda + m_A^2 A_+^\lambda)\,(\bar{e}\gamma_\lambda(1+\gamma_5)\,\nu_e),$$

we are led to the effective nuclear beta decay interaction

$$\frac{G}{\sqrt{2}}\left(\bar{N}\left\{\gamma^\lambda\left(1-\frac{t}{m_v^2}\right)^{-1} + \frac{5}{3\sqrt{2}}\,\gamma^\lambda\gamma_5\left(1-\frac{t}{m_A^2}\right)^{-1}\right\}\tau_+ N\times\right)$$
$$\times\cdot(\bar{e}\gamma_\lambda(1+\gamma_5)\,\nu_e).$$

This gives a ratio for the Gamow–Teller and Fermi interactions of

$$g_A = 5/3\sqrt{2} \simeq 1.2.$$

The interaction structure also predicts a certain amount of weak magnetism and a certain amount of CP violation in weak axial vector interactions. The latter is a weak electric dipole estimated to be of the order

$$(f'/f)\cdot(m_e/m_A) \simeq 10^{-3}$$

in the ratio of the amplitudes, completely beyond experimental detection at the present time.

Before estimating the weak magnetism term we could reassure ourselves that this is a legitimate method and derive some useful results by applying the same considerations to the electromagnetism of the nucleons. We postulate that electromagnetism is an induced property for the nucleons, the primary electromagnetic interaction being of the form

$$\left(\frac{e}{g}\right)(m_v^2 V_0^\lambda + m_w^2 W^\lambda)\,A_\lambda$$

where A_λ is the electromagnetic four-potential and W^λ is a neutral vector meson field. To arrive at the ratio of the strong coupling constants we study the $SU(4)$ invariant coupling of the meson matrix made up of V_α^0 ($I=1$, $J=0$), $\frac{1}{m_\phi}\nabla\times\boldsymbol{\phi}$ ($I=0, J=1$) and $\frac{1}{m_v}\nabla\times\mathbf{V}_\alpha$ ($I=1, J=1$). We get

$$(g'_\infty/g) = 1; \quad (g'/g) = (5/3).$$

We can now use these coupling constants to deduce the nucleon magnetic moments. We get:

$$\mu_p = 1 + \frac{5}{3}\cdot\frac{m_N}{m_v} = +3.0$$

$$\mu_n = -\frac{5}{3}\,\frac{m_N}{m_v} = -2.0$$

These results compare favourably with the physical results.

Reassured by these results we can return to the calculation of the weak magnetism terms. We obtain a contribution of the form

$$\frac{G}{\sqrt{2}}\,\frac{\varkappa}{2m_N}\,(\bar{N}\sigma^{\lambda\nu}q_\nu\tau_+N)\,(\bar{e}\gamma_\lambda\,(1+\gamma_5)\,\nu_e)$$

with

$$\varkappa = \frac{2m_N}{m_v}\cdot\frac{g'}{g} \simeq 4$$

in good agreement with the experimental result.[2]

Let us recapitulate what we have done so far: the departure of the nucleon weak interaction from the chiral V–A form has prompted us to postulate a two-step interaction structure, the primary weak interactions being the coupling of vector and axial vector mesons with the leptons. We are able to derive a very satisfactory effective beta decay interaction. In the process we have obtained a very detailed structure for the coupling of the vector and axial vector mesons with the nucleons.

We might ask ourselves if the strong interaction so obtained is itself a satisfactory one. Both pion–nucleon and nucleon–nucleon interactions have been analyzed on the basis of this primary interaction. The results are very gratifying. These comparisons are too extensive to reproduce here but they are already available in the published literature[1,3,4]. We note, in passing, that the conservation of the source of vector mesons implies that no scalar mesons are to be included; but the axial vector meson source is not conserved and hence there are pseudoscalar mesons. We may write:

$$\partial^{\mu} A_{\mu} = \xi \cdot m_{\pi} \cdot \phi_{\pi},$$

where ξ is a suitable numerical parameter. Comparison with experiment yields

$$\xi \simeq 0.1$$

II A MODEL FOR SEMILEPTONIC MESON DECAYS

When we come to strange particle weak interactions we find that their weak interactions which involve change in strangeness seem to be always suppressed by an order of magnitude. We also note that while the strange particles clearly fall into $SU(3)$ multiplets, among the members there are significant mass differences. As a consequence, in any relativistic algebraic scheme the momentum operators P^{μ} cannot commute with the $SU(3)$ operators. A simple scheme of obtaining a useful algebra is to make P^{μ}/M commute with the internal symmetry generators. This scheme has the advantage that a particle at rest is connected only with other particles at rest by the symmetry generators. The mass formula is contained in the detailed specification of the nonlinear relation between the Poincaré and the internal symmetry generators.

An algebra of this type, called $\mathscr{B}$, has been developed to deal with the mass spectrum of hadrons.[5] We have made an algebraic model for weak interactions of mesons[6] based on the algebra $\mathscr{B}$.

We start with a transition operator for semileptonic transitions of the form

$$T = G \int d^4x\, L^{\lambda}(x)\, H_{\lambda}(x)$$

$$H_{\lambda}(0) = \sum_{a=1,2} \{P_{\lambda}, E_{\alpha} + F_{\alpha}\}$$

$$\langle \bar{\nu} e \alpha_1 \quad \alpha_N p_{\nu} p_e p_{\alpha_1} \cdots p_{\alpha_N} | L^{\lambda} | \gamma q_{\lambda} \rangle$$
$$= (ab)^{N+1}\, \delta^{(3)}(p_{\nu} + \cdots - p_{\gamma})\, \bar{u}_e \gamma^{\lambda} (1 + \gamma_5)\, u_{\nu}.$$

Here b is an invariant normalization constant which drops out in the final calculation and a is a parameter which can be computed from the branching ratio $(\pi \to \pi\, e\nu)/(\pi \to \mu\nu)$. The details of the calculations are too lengthy to be reproduced here; they are available elsewhere in the literature. We shall only quote the main results and their comparison with experimental results.[7]

i) The application to pion and kaon decays leads to the suppression factor

$$S_{l2} = \sqrt{\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} \Big/ \frac{m_\pi}{m_K} \cdot \frac{1 - (m_\mu/m_\pi)^2}{- (m_\mu/m_K)^2}} = \tan \theta_A$$

with the Cabibbo suppression factor

$$\tan \theta_A = \frac{m_\pi}{m_K} \cdot \frac{1 + (m_\sigma/m_K)}{1 + (m_\sigma/m_\pi)} = 0.28$$

to be compared with the experimental value 0.27.

ii) For the three-body decays we get the suppression factor

$$S_{l3} = \frac{m_\pi}{m_K} \frac{1 + (m_\pi/m_K)}{1 + (m_\pi/m_\pi)} = 0.18$$

to be compared with 0.21

iii) For the conventional K_{l3} decay matrix element written in the form

$$\frac{G_K}{\sqrt{2}} \{(f_+ + f_-) p_\lambda^{(K)} + (f_+ - f_-) p_\lambda^{(\pi)}\} \bar{u}_e \gamma^\lambda (1 + \gamma_5) u_\nu$$

we obtain

$$f_+(q^2) = f_-(q^2) = 1 + \lambda q^2/m_\pi^2 = f_+(0)$$

so that

$$\lambda = 0; \quad \xi = f_-/f_+ = +1.$$

iv) For the K_{l4} decay we predict an absolute decay rate of $3.47 \times 10^3\ \mathrm{sec}^{-1}$ while the experimental value is $2.9 \times 10^3\ \mathrm{sec}^{-1} \pm 20\%$.

v) The decay $K^0 \to \pi^0 \pi^- e^+ \nu$ is forbidden.

vi) We predict

$$\Gamma(K_{\mu 4})/\Gamma(K_{e4}) = 0.17$$

to be compared with the experimental value 0.39 ± 0.26.

It thus appears that the separation of the semileptonic meson processes into two stages, one governed by standard matrix elements and the other related to an algebraic model for mesons is a useful one.

III SEMILEPTONIC DECAYS OF BARYONS

The baryons are not connected to the mesons by the algebra $\mathscr{B}$. We may therefore explore a different scheme for their interactions somewhat similar to the one we employed for deducing the nucleon weak interaction properties. We shall, accordingly separate the interaction into two stages, one in which the baryon transmutes itself into a baryon and a vector or axial vector meson; and the other in which the meson couples to leptons. We shall use the standard coupling

$$\left(\frac{G}{g}\right)(V^{\lambda}m_v^2 + A^{\lambda}m_A^2 + V'^{\lambda}m_{v'}^2 + A'^{\lambda}m_{A'}^2)\,(\bar{e}\gamma_\lambda(1 + \gamma_5)\,\nu_e)$$

for both strange and nonstrange mesons. The observed weak semileptonic effects depend on the details of the strong interaction.

We hav already seen the relation between the nonstrange axial vector and pseudoscalar fields:

$$\partial_\lambda A^\lambda = \xi m_\pi \phi_\pi.$$

Let us similarly set

$$\partial_\lambda A'^\lambda = \xi m_K \phi_K.$$

We can immediately recalculate the rates of pion and kaon decays from these relations. Agreement with the Cabibbo factor $\tan\theta_A = m_\pi/m_K$ is obtained if and only if the parameter ξ is the same for strange and nonstrange mesons.

It is now well established that not only do the baryons and pseudoscalar mesons fall into octets, but their coupling is in accordance with $SU(3)$ invariance. We do not have any such immediate evidence for either the axial-vector mesons or the vector mesons. In fact, if the PCAC relations above are to be maintained, the axial vector mesons cannot be coupled according to $SU(3)$. For example, for the ΛpK vertex, the axial vector coupling constant should be $\left[(3-2d)/\sqrt{6}\right]$ times the $pp\pi$ vertex (where d is the D/F ratio) according to $SU(3)$. But the corresponding axial vector coupling constant to the Λp vertex is

$$\left\{\frac{2}{1+\dfrac{m_\Lambda}{m_N}}\cdot\frac{m_\pi}{m_K}\cdot\frac{3-2d}{\sqrt{6}}\right\}$$

times the coupling to the pp vertex. Consequently the axial vector leptonic decay of the Λ hyperon is suppressed by a Cabibbo factor:

$$\tan \theta_B = \frac{2m_\pi}{m_K \quad + \frac{m_\Lambda}{m_N}\Big)} \simeq 0.26$$

This is in good agreement with experiment.

For the nonstrange vector mesons we had demanded that their sources were divergence-free. We make the same demand of the sources of the strange vector mesons. But now because of the large mass differences between the baryons, this forbids the direct (vector) coupling of the strange vector mesons. They can couple only by tensor coupling. This would mean that the vector decays would be suppressed below the $SU(3)$ expectations though the modification cannot be expressed as simply as assigning a Cabibbo factor. This hypothesis ought to be tested experimentally.

REMARKS

We have discussed three distinct but interrelated algebraic models for semileptonic weak interactions of hadrons. They deal with nuclear beta decay, semileptonic decays of mesons and semileptonic decays of baryons respectively. In the course of this we have established some satisfying connections with strong and electromagnetic interactions. A number of predictions are obtained which yield good agreement with a variety of experimental results. Further study and a synthesis of these various models seem to be indicated.

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DISCUSSION

Raman If the experimental $f_-^{(0)}/f_+^{(0)}$ changes from unity could you modify your theory?

Sudarshan No, the value unity comes from the basic structure of the interaction. Rather than twist the model to reproduce the experimental number, I would simply state that if it were the case, the model disagrees with experiment.

Gunzik Could you extend your models to nonleptonic decays?

Sudarshan No; since I deal with the leptons explicitly the present model must have leptons.

Lipkin How does this additional coupling with the spinor fields affect PCAC and the Adler–Weisberger relation?

Sudarshan The PCAC is totally unaffected because the additional term does not change the divergence. The Adler–Weisberger relation is a little difficult to explain right now because I don't know how to relate the purely abstract commutation relation to the present formulation except to say that both of them give the same final result.

Breit It seems that for many years the nature of the proton magnetic moment and the neutron moment have been a kind of a triumph of many theories. It is most impressive to see so many good pre[illegible]. But it is in contrast with the philosophies that we were all talking about in the past. In a case like the protons and neutrons, it is hard to believe that the way in which they behave and the way in which they are positioned in space are regulated by geometry. Now perhaps I am a little old-fashioned in thinking that a geometrical picture is not a likely picture.

Sudarshan I would certainly not dare take exception to your views about these matters. I would like to mention, just as a curio, an item, which I hope will put my attitude towards the present theory in the proper setting! Many years ago when I was a student with Professor Marshak studying the anomalous magnetic moments of the nucleons I decided to do some literature search. I found that the best prediction to that date on the proton and neutron magnetic moments was made by Eddington in his *Relativity Theory of Protons and Electrons*; he had fantastically good numerical predictions. So I also would not be willing to judge a meson theory merely by its ability to predict proton and neutron moments.

SNOW With respect to Hyperon–Semileptonic Decays, does your theory predict significant differences from the simple Cabibbo theory with the same angle for Vector and Axial Vector decays?

SUDARSHAN Not for the Axial vector decays since I get the octet transformation property and $\tan\theta_A = m_\pi/m_K$. But I do not have a similar situation in the "vector" decays since the hyperon matrix element is purely a Pauli type coupling without any pure vector terms in it.

YODH George, I would like not to question the elegance of any theory, but would like to know how many parameters did you use and how many pieces of data did you fit on the two models? I would like to ask this question for any theory that is presented to get a feeling for how many assumptions one has to make.

SUDARSHAN For the fits to the neutron and proton moments for weak magnetism and for g_A *no* parameters. In the second model I use a and b obtained by fitting $\pi^+ \to \mu^+\nu$ and $\pi^+ \to \pi^0 e\nu$ rates. The $f_-^{(0)}/f_+^{(0)}$, the absolute K_{l3} and K_{l4} rates, the $K_{\mu3}/K_{e3}$ ratio, the momentum dependence of the $K_{\mu3}$ form factors and the forbiddenness of the K_{e4}^0 and the $(\pi \to \mu\nu)/(K \to \mu\nu)$ are all genuine predictions.

TODOROV Concerning the number of parameters you have at your disposal, I would like to note that there is an arbitrariness in choosing $H_\lambda(0)$ to be $\{P_\lambda, E_\alpha + F_\alpha\}$. Since this expression is not $SU(3)$ invariant the relative coefficients in front of the different components of the eight vectors E and F are independent parameters. To take all these coefficients equal to 1 is a $SU(3)$ basis dependent choice which is as arbitrary as any other, but this is not a natural thing to put because it is not a covariant expression.

SUDARSHAN I'm sorry, but this is an explicitly Poincaré invariant theory

TODOROV I had in mind, covariance with respect to the internal symmetry, and this is not a very natural summation.

SUDARSHAN Not a summation of all the indices α. There are only two of them taken with equal weight, very much like the electron and muon currents are taken with equal weights.

FRANKLIN I don't see any reason why the derivative coupling should equal the nonderivative coupling. Is there any reason?

SUDARSHAN To form the $SU(4)$ multiplet for the $I = J = 1$ state we have the choice of $\mathbf{A}$ or $\frac{1}{m_A}(\nabla A^0 - \dot{\mathbf{A}})$. Since there is no a *priori* reason to choose one or other form of coupling, we have chosen them with equal weight. We verify that this is the proper choice by computing the value of g_A obtained from this. Chiang, Gleisher, Huq and Saxena (*Phys. Rev.* **177**, 2167 (1969) have shown that the coupling constants so obtained give good no-parameter fits to the nucleon–nucleon scattering data.