IN SEARCH OF SCHRÖDINGER

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His cat in the famous thought experiment was both dead *and* alive—until the wave function collapsed—that is. In this tribute we prove that the fate of one of the founders of quantum mechanics is very similar

ERWIN Schrödinger is one scientist whose name is identified with the very discipline of quantum physics and chemistry. The equation which bears his name is the basic equation of atomic physics and of the other areas of quantum mechanics. Schrödinger's work brought out the subtle connection between discreteness characteristic of the 'quantum' nature of matter and radiation and the smooth behaviour of solutions of wave equations. Schrödinger also showed that his epoch-making formulation of 'quantization as an eigenvalue problem' and Heisenberg's formulation of matrix mechanics were equivalent. For these contributions Schrödinger was awarded the Nobel prize in physics in 1933; he shared the prize with Paul Dirac, the great architect of quantum mechanics and so much else in modern physics.

The discovery of the quantum

At the turn of the century, Max Planck had shown that if radiant energy was thought of as discrete parcels of energy or 'quanta', then not only could the contradiction between the two well-established classical physical theories of statistical thermodynamics and electrodynamics be removed, but one could get a very quantitative understanding of the distribution of radiant energy from a heated cavity. The corresponding spectrum called the Planck radiation law is familiar to all of us who have gazed at a steady fire, whether it be in a fireplace or a *homa kunda*: in the centre of the fire all bodies are turned into a golden glow which changes as the temperature varies.

Five years later, Albert Einstein showed that photoelectricity obeyed laws that could be understood on the basis of quanta. Five more years later, Niels Bohr used quantum ideas to explain the main features of the hydrogen spectrum. Six years later, Peter Debye used quantum ideas to explain quantitatively the behaviour of the specific heat of certain solids at very low temperatures: when one reached very low temperatures, solids seemed to heat up with very little heat added to them! Despite all these successes, the quantum remained an alien: physics did not have a logical basic way for the quantum to be incorporated. It was always brought in by ad hoc reasoning.

Meanwhile, evidence for quantum aspects of radiation and matter kept mounting. X-rays were discovered and characteristic X-ray spectra of atoms determined. Arthur Compton showed that X-rays suffered a change in their wavelength on being scattered by atoms. This Compton effect suggests that X-rays travel as quanta and collide elastically with electrons.

The clue to taming the wild quantum came with the work of Louis de Broglie and his matter waves. He felt that if radiation which appeared continuous had the graininess of quanta, then matter which appeared granular in sub-atomic particles like electrons must behave as waves. Subsequent experiments verified that this was indeed so. The crisis deepened: What is grainy is smooth and what is smooth is grainy at the fundamental level!

The mystery was strengthened by Satyendranath Bose who showed that the quanta of light considered as particles obeyed a new statistics, a new rule for counting distinct states from ordinary particles. Bose went on to show in the same year, 1924, as de Broglie, that these rules led naturally to Planck's spectral law.

Schrödinger installs the quantum

The connection between continuity (smoothness) and discreteness (graininess) had been known in mathematics and specially discussed by David Hilbert. If we take a smooth ball of dough and subject it to a homogeneous strain we get an ellipsoid. (This happens when professional scientists turn amateur cooks and try to make *chappattis.*) If we look at the special directions corresponding to the axes of the ellipsoid, apart from a rotation, we have a simple compression which however, is different in different directions. These are the 'proper' directions and the magnitude of the compressions are the 'proper' values.

Instead of considering a three-dimensional body and its transformation, if we considered the transformations in infinite dimensional space, we could ask if similar ideas work. Since we are now in infinite dimensional space, great care must be exercised but the result is that when such care is exercised we could deal with 'eigenvalues' (proper values). A special example of such a space is given by (complex) functions whose (absolute value) squared integrated is finite.

Schrödinger showed in 1926 that this mathematical connection between eigenvalues and transformations of wave functions could serve to connect the discrete quantum and the con-

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ERWIN SCHRÖDINGER AS LIFE SCIENTIST

ERWIN Schrödinger's was surely one of the liveliest minds. More than a peep becomes possible into it through his two 'little' books, *What is Life?* and *Mind and Matter* based on the lectures given at Trinity College, Dublin in 1943 and in 1956 respectively. His objective in these lectures was to overcome the constraints imposed in the viewpoint of life by any scientist's specialization and to range widely like the ancient Greeks. In 1943, the momentous paper of Avery and colleagues was still a year in the future, in which for the first time a hereditary role for DNA (deoxyribonucleic acid) was discerned. Small wonder that Schrödinger also assumed, as did the biologists of the time, that genetic material must be proteinaceous. The catalytic functions in a living cell are mostly performed by the enzymes which Sumner had showed are also proteins. So it seemed like a legitimate mistake that with their versatility proteins should also be the hereditary material.

Physicists had been quite intrigued by the (relative) 'permanence' of the hereditary material. A dominant mutation 'Habsburg' lip—the fleshy lower lip and protruding jaw running in the Habsburg royal line—had been traced through family portraits back to their fifteenth century ancestors. The gene governing this character had obviously been transmitted unchanged over hundreds of years through several generations. Schrödinger wondered: 'How are we to understand that it has remained unperturbed by the disordering tendency of the heat motion for centuries?' The conclusion seemed inescapable to Max Delbruck and to Schrödinger: the genes must be molecules.

It was already known that genes are part of the microscopically visualizable fibre-like structures, chromosomes. In fact, for the giant salivary gland chromosomes of *Drosophila*, in which thousands of darkly stained 'transverse' bands can be seen, C. D. Darlington had proposed that each band corresponded to a gene. So what was the size of the gene? From radiobiological experiments, it was estimated that the target size of a gene may correspond to about 1,000 atoms. In one of the most remarkable predictions, Schrödinger reached the conclusion that chromosome fibre may suitably be called an *aperiodic* crystal and its large number of unit combinations would provide the basis for hereditary code. Although this idea turned out to be correct in a general way, in concrete terms perhaps it was less true because Schrödinger tried to explain, as did Delbruck, mutations in terms of isomerism (rearrangement of atoms) rather than, as it turned out to be in terms of alteration of nucleotide sequence. But there was no getting away at that time from the beguiling quantum theory and Schrödinger explained mutation as a quantum jump.

While *What is Life?* is often talked about, Schrödinger's *Mind and Matter* is less known, one explanation for which may be that DNA had in 1956 already made a firm debut, its proposed double helical structure providing a successful basis for the key genetical functions. Mind is a much more complex system than hereditary apparatus. In spite of this, Schrodinger provided an interesting insight or two. His general hypothesis: 'consciousness is associated with the *learning* of the living substance; its *knowing how (Konnen)* is unconscious.' After ploughing through some heavy stuff which also does not seem to have relevance for the present times, one finally reaches the last chapter of the book: The mystery of the sensual qualities. This is somewhat interesting from the point of how we perceive things.

Schrödinger takes off from Democritus of Abdera, an ancient Greek, who made a distinction between the intellect and the senses. In an argument about what is 'real', the intellect says, 'ostensibly there is colour, ostensibly sweetness, ostensibly bitterness, actually only the atoms and the void' to which the senses reply: 'Poor, intellect, do you hope to defeat us while from us you borrow your evidence? Your victory is your defeat.' Schrödinger provides interesting examples. A physicist, for example, considers yellow light as only transversal electromagnetic waves of 590 millimicron (mμ) wavelength. How does the yellow come in? These kind of vibrations hit the (human) retina and give the person a sensation of yellow. Different wavelength radiation produces different colour sensations but only in the region of 400 and 800 millimicron. Why only this spectrum? Apparently due to an adaptation to the solar radiation which is strongest in these wavelengths. Sun's radiation is maximal at 590 mμ which gives intrinsically the brightest sensation.

Intriguingly, if waves of 760 mμ (red) are mixed in a definite proportion with those of 535 (green) this produces a yellow indistinguishable from yellow produced by 590 mμ. Mixture of 'red' and 'blue' gives 'purple', a colour not produced by any monochromatic light. The sensation of the colour cannot be explained by the physicist's objective picture of light waves. Even a physiologist may not have a good explanation for the sensation. Thus an observer's colourful impression may not provide clue to its physical nature. That can only be done by instruments. The second conclusion is that the observer is never fully replaced by the instrument. If he were, he will know nothing new. Schrödinger reaches the enigmatic conclusion that while senses tell us very little about the objective physical nature and yet the physical picture rests in a complicated way derived by direct sensual perception.

One may facetiously conclude then that if man is a handiwork of God, then God must be a biologist since in introducing several caprices (like inverting of the image or sensation of the yellow) in the human mind, He has evaded the physical laws!

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tinuous wave. In a series of papers, Schrödinger developed the idea of 'quantization as an eigenvalue problem'. In one stroke he connected the quantum with familiar differential equations of physics, and at the same time gave a familiar mathematical procedure for calculating the quantum energy levels of a system like the hydrogen atom. Schrödinger's papers are surprisingly clear and direct and of use even today.

Having done this, it was only natural that Schrödinger could see that the agreement between the results of his method of the wave equation and Werner Heisenberg's method using noncommuting operators was not a coincidence but a deep structural identity: he gave the formal proof within six months! In this proof of equivalence we are told that Schrödinger had the benefit of the counsel of Hermon Weyl, mathematician and natural philosopher.

A *rishi* in science

The history of the development of quantum theory in the first four decades of this century is sparkling with many illustrious names who made significant and lasting contributions. Many of these were timely and influenced by the proximate experimental and theoretical work. But out of these people Schrödinger and Dirac stand out in that they seem to be seers whose vision could far transcend their times and who seemed to chart a course of discovery and insight on their own. In short, I am tempted to call them *rishis* of modern physics.

Schrödinger contributed to several areas of physics and beyond. In each, his perception and presentation is clear, simple and persuasive. His book *Space-time Structure* is a treatment of general relativity and gravitation, but proceeds by first developing the tools of first affine geometry and then introducing the notion of a metric. An affine geometry is one in which we can make parallel transport of vectors, that is define a rule by which we can define 'the same direction'. For a flat space like the one in which we do Euclidean

geometry this appears trivial, on a curved surface like the surface of an orange this is no joke. (After all the idea of thinking of 'the vertical' all over the surface of the Earth is enough to introduce vertigo for us!)

To define this part of the structure we do not need to define 'the metric', the notion of distances. But usually one introduced first a general metric space and then we defined the affine structure. That is, first the notion of distance is introduced and then the notion of parallelism. Schrödinger inverted this order and thus showed the logical independence. What is more, it simplified the logical development of the theory. Formally, it was most prophetic in that the notion of affine structure gets extended when we deal with gauge theories which contemplate symmetry transformations at different places. The structure of such theories is anticipated by Schrödinger's development of general relativity in *Space-time Structure*. The cosmological theory then understood is well exposed in *The Expanding Universe*.

Schrödinger's book on statistical mechanics is similarly a masterly exposition of the essential principles underlying equilibrium statistical mechanics. Many of the standard books got involved in specific model calculations, detailed computations and justifications for the various approximation schemes to estimate integrals of interest in the context of statistical mechanics. In all this sound and fury the principles of statistical mechanics often got lost. If you did not know what were the defining principles, you were not likely to know by the study of these heavy books. In contrast, in Schrödinger's slim volume *Statistical Mechanics* there is the development of the fundamental principles and you are then told that all else is your problem and you may solve it in whatever way you choose. The so-called partition function defines the 'free energy' from which the entire thermodynamics may be deduced. Simple calculations are done to recapitulate known simple results and that is all.

The most influential of Schrödinger's books is his slim volume titled *What is Life?*, a systematic search into the problem of life and an enquiry into the possibility whether physical laws can generate, sustain and propagate life. This book was first published in 1944 based on a series of lectures given by him the previous year at Trinity College, Dublin. It has been a best seller and has been read by scientists and non-scientists for the past few decades.

The book begins with the comparison of the orderliness of living systems with the orderliness of physical systems and then proceeds to review formal genetics. He then goes on to bring out that the reproductive invariance and occasional gene mutation requires the operation of quantum theory, that the gene is a macromolecule. He goes on to argue that unlike a crystal which could be obtained by repeating a periodic pattern, an aperiodic structure may embody 'a complicated system of determinations within a small spatial boundary.' This is a clear anticipation of the genetic code.

Schrödinger had been deeply concerned about 'order'. The fact that living systems produce order by living in a non-equilibrium open system is in itself not surprising nor specific to living systems. But there is the remarkable fact that the genetic information coded in the gene as a structure is transcribed and governs the processes, a sequence of chemical reactions with fine accuracy. Schrödinger was aware and did elaborate on this negentropic and order generating aspects.

Schrödinger reflected into realms which most scientists are shy to enter

But the most remarkable books of Schrödinger, *Mind and Matter* and *My World View*, are reflections by an accomplished scientist into realms which most scientists are shy to enter. Here he talks about consciousness and perception, the problem of subjective and objective knowledge and the problem whether consciousness is singular or plural. Schrödinger's views are rather unusual for an Austrian scientist in the early twentieth century and more akin in spirit to a *vedantin*. But apart from his views, his treatment of the problem is so masterly and his argumentation so persuasive that one is amazed. One can only marvel at the ability of someone like him to transcend the cultural persuasions of his times and surroundings. I wonder how many amongst us could have this ability?

For all his brilliance Schrödinger seems not to have left a school behind him. A part of this may be due to his being so isolated for the latter part of his life; part due to his being so self sufficient. He seems not to have gained the stimulation of other experimental physicists or biologists. One has heard of stories of how Schrödinger was to have come permanently to Bangalore and wonders how much it would have altered science in India and Schrödinger's own work. With people like C. V. Raman and H. J. Bhabha to stimulate and be stimulated by him and people like S. S. Bhagavantam, B. S. Madhava Rao and Harish Chandra full of ability to do theoretical physics and G. N. Ramachandran to do biophysics, it could have been a glorious decade or two.

I have already remarked on the *rishi*-like insights of Schrödinger and of Dirac (though perhaps it is sacrilege to compare anyone with Dirac!). If one contrasted Schrödinger with Albert Einstein (another sacrilege!), we see two very different kinds of people. Einstein, at least for three decades, followed every important development in modern physics in both experimental and theoretical work; examined the relationship between them, recognized the major advances and contributed significant works of his own. Many a time Einstein was able to abstract the essence from already known results, add his modifications, and present it to be accepted by the world as if it were entirely a new theory that Einstein gave. Schrödinger on the other hand, did his own work and followed his own counsel. It appears that consequently Schrödinger's influence on physics was less than it could have been. But then each must follow his *svabhava*.

Schrödinger's later writings are in English and it is remarkable how felicitously he writes. It is all the more remarkable when we recall that English was not his first language. Among the modern masters only my mentor Julian Schwinger seems to write with such felicitousness and such conviction. But while Schwinger compels, Schrödinger persuades; and with either of them, if you don't watch out, you are seduced.

When I look over and marvel at Erwin Schrödinger as a creative writer and as a scientist, I see more in common between him and the *rishis* of the *upanishadic* times than with the majority of twentieth century writers of literature or science. Perhaps times will change, and conviction and vision will again receive admiration and following.

ERWIN SCHRODINGER: SCIENTIST

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