Conserved Currents in Weak Interactions*

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Yakov B. Zeldovich was one of the most original and versatile theoretical physicists in recent decades. His bold and imaginative speculations (particularly in particle physics and cosmology) may have appeared eclectic at times, but only because the rest of us took longer to appreciate their depth and innovativeness. His early and sudden death in December 1987 has greatly saddened his many admirers and friends. The authors of this tribute to Professor Ya. B. Zeldovich try to place his work (with S. S. Gershtein) on the conserved vector current hypothesis – proposed more than 30 years ago – within the broader framework of the development of weak interaction theory.

Gershtein and Zeldovich's CVC hypothesis in weak interactions took its cue from quantum electrodynamics, and it is amusing to note that there has been an increasingly close interplay between the electromagnetic and weak interactions ever since beta rays and gamma rays were discovered at the turn of the century. While the electromagnetic interaction is, in principle, the same as in classical Maxwell–Lorentz systems, the quantum theory of gamma rays should be traced to Dirac's treatment of the semiclassical theory of radiation using the interaction picture.[1]

Fermi's elegant recapitulation of Dirac's theory in his 1932 review paper on "Quantum Theory of Electromagnetic Radiation"[2] set the stage 2 years later – after Pauli had proposed the neutrino to save the conservation laws in nuclear beta decay – for the first explicit (second-quantized) formulation of a theory of weak interactions[3] patterned after the electromagnetic interaction. Fermi selected the vector (V) interaction out of five possible choices [the others being scalar (S), pseudoscalar (P), axial vector (A) and

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tensor (T)] in analogy to the electromagnetic interaction even though the analogous "current" in β decay was charged and not neutral. There was no mention of a relationship between the currents of electricity and of weak interactions, and Fermi applied his vector theory mainly to non-relativistic nucleons and the calculation of spectra and total transition rates. Parity conservation, as well as baryon and lepton conservation, were implicitly assumed in the Fermi theory. In calculating the differential beta spectrum for the allowed case, Fermi took account of the Coulomb effect of the nucleus by replacing the electron wavefunction by its value at the nuclear radius with the result:

$$\mathrm{d}N_e = [G^2/(2\pi)^3]pE(E_0 - E)^2 F_0(Z, E)\,\mathrm{d}E, \tag{1}$$

where G is the coupling constant, E, p the electron energy and momentum, E_0 the maximum energy, and $F_0(Z, E)$ the Coulomb function. The nuclear matrix element is taken to be unity in eq. (1). More generally, one must include an additional factor $|\int 1|^2$ for this matrix element.

In the non-relativistic approximation for the nucleon, V and S couplings predict the same nuclear spin and parity changes in an allowed β transition, namely $\Delta J = 0$, no parity change (Fermi selection rule). Within a short time, Gamow and Teller[4] pointed out that the β interaction can depend on the spin of the nucleon (as contained in the T or A interaction) and, in that case, the selection rule is: $\Delta J = 0, \pm 1 (0 \to 0$ transition is forbidden) and no parity change, for an allowed transition. A prime example in support of the Gamow–Teller conjecture was He^6 $(J = 0^+) \to$ Li^6 $(J = 1^+) + e^- + \bar{\nu}_e$, which decay played such an important role in the quest for a universal β interaction in later years. Hence, the distinction between the Gamow–Teller selection rule (corresponding to the A or T interaction) and the Fermi selection rule (coresponding to the V or S interaction), is essential.

The observation of Gamow–Teller β transitions implied that Fermi's V interaction could not be the sole β interaction, and might even be absent. The precise structure of the β interaction immediately became a burning question, and a variety of methods was suggested to determine its form. Thus, Fierz[5] pointed out that the presence of S *and* V or T *and* A in the β interaction leads to an interference term in the allowed β spectrum which vanishes in the absence of an admixture of S *and* V or T *and* A. Thus, the inclusion of Gamow–Teller interactions modified the differential transition rate to the form:

$$\mathrm{d}N_e = \frac{1}{(2\pi)^3}\left(G_F^2\left|\int 1\right|^2 + G_{GT}\left|\int \sigma\right|^2\right)pE(E_0 - E)^2 F_0(Z,E)\left(1 + \frac{bm_e}{E}\right)\mathrm{d}E. \tag{2}$$

where $(1 + bm_e/E)$ is the Fierz interference factor[5] that may come from interference between S and V *or* T and A interaction. Measurements were soon in hand[6] with limits on b: $b = 0.02 \pm 0.09$ for a Fermi transition and

$b = -0.007 \pm 0.010$ for a Gamow–Teller transition. These results strongly supported the idea that there was no mixture of the S and V interactions *or* of the T and A interactions; nothing could be said about the P interaction, which has no non-relativistic limit for the nucleon. It should be understood that in all β-decay calculations, the relativistic expressions for the S, V, A, T, P forms of the electron–neutrino current are used in the β-decay interaction.

With improvements in experimental technique in the early 1950s,[6] it was possible to undertake electron–neutrino angular correlation experiments in β-decay. It is easily shown that, at any fixed electron energy, the correlation function (for allowed decays) is of the form:

$$\left(1 + \lambda \frac{v}{c} \cos \theta_{e\nu}\right) \tag{3}$$

where the coefficient λ has different values for S, V, T or A, as given in Table 1.

The first serious electron–neutrino correlation experiment was undertaken with He^6 because it was clearly a pure Gamow–Teller transition. The measurement of $\lambda = 0.33 \pm 0.08$ in He^6 decay[7] strongly favoured T as the Gamow–Teller contribution to the β interaction since the A interaction requires λ to be -0.33, many standard deviations away. If the T interaction was so clearly favoured by the electron–neutrino angular correlation experiment in He^6, one could only accept two possible combinations of the β interaction, namely S *and* T *or* V *and* T (except for a possible admixture of P). It should be emphasized that this was the state of affairs before the Gershtein–Zeldovich paper was published – in 1955 – and before parity violation was discovered in β-decay – in late 1956.

Before we turn to the conserved vector current paper of Gershtein and Zeldovich, we must take note of the discovery of the strongly interacting pion and the "second-generation" muon in 1947.[8,9] The discovery of the pion made it clear that the presence of the nucleons inside a nucleus in β-decay subjected them to strong nuclear forces resulting from the emission and absorption of pions, which could, in general, modify the matrix elements for any postulated Lorentz structure of the nucleon

TABLE 1. *Values of the* (e, ν) *angular correlation coefficient* λ *in allowed transitions*

Interaction	λ
S	-1
V	$+1$
T	$+1/3$
A	$-1/3$

current. It seemed that these strong interaction effects would introduce a new element of uncertainty into the theory of β-decay. On the other hand, the discovery of the muon had more promising ramifications: it greatly increased the number of weak interaction processes that could be studied, as can be seen from Figure 1. Within a couple of years (1947–49), the famous "triangle" of weak interactions (see Figure 1) was used to test the universal Fermi interaction (UFI), i.e. the hypothesis that all weak interaction processes were of equal strength. Let us recall that Pontecorvo[10] had noted the rough equality $g_3 \sim g_1$ (Figure 1) and Marshak and Bethe[9] had related $g_{\pi NN}$ and $g_{\pi\mu\nu}$ to g_3 but muon decay had not as yet been brought into the discussion. During 1948–49 a number of authors[11] examined the relationship of the various weak processes implied by Figure 1, with and without the mediation of the strong pion–nucleon interaction. The overall conclusion was that muon decay fitted into the UFI hypothesis (i.e. $g_2 \simeq g_1$) as long as the β interaction was not predominantly P. It was not until the universal V–A Lorentz structure was established for all weak processes beginning in 1957 (see below), that UFI received definitive confirmation.

One very useful observation resulting from the early UFI discussion was that of Ruderman and Finkelstein,[12] who noted in 1949 that the ratio of the decay rates of the pseudoscalar π into (e, ν) and (μ, ν), namely $\mathbb{R} = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$, is *independent* of the strong pion–nucleon interaction. If one assumes electron–muon universality (i.e. $g_1 = g_3$), $\mathbb{R}$ depends only on the form of the weak coupling in the following fashion: $\mathbb{R} = 1.2 \times 10^{-4}$ for the A weak interaction, $\mathbb{R} = 5.4$ for the P weak interaction and $\mathbb{R} = 0$ for the S, V or T weak interaction. Five years later, Finkelstein and Moszkowski[13] returned to the related questions of UFI

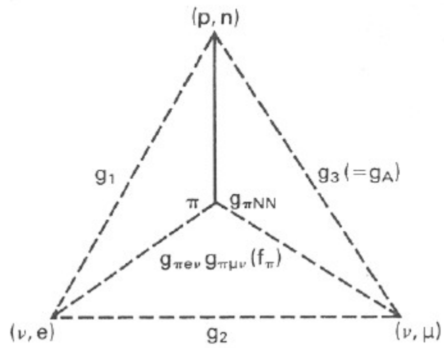

FIGURE 1. Triangle of weak interactions: diagrammatic sketch showing the weak interactions (dotted lines) and the strong interaction (solid line).

and the Fermi type and Gamow–Teller type coupling constants, g_F and g_{GT} respectively. The analysis of available *ft* values for the mirror–nuclei transitions n–p, He^3–He^3, O^{15}–N^{15} and F^{17}–O^{17}, led them to determine $g_{GT}^2/g_F^2 \simeq 1.6 \pm 0.2$. They attempted to attribute this effect to the vertex modification brought about by virtual emission and reabsorption of neutral pions. They deduced that the sign of the effect was correct, and that the magnitude was constant with the parameters of the Chew static cut-off theory;[14] they concluded that the

> existence of this mesonic perturbation of the correct sign and approximately right magnitude makes it possible to assume that the unperturbed Gamow–Teller and Fermi constants were exactly equal in accordance with various hypotheses about the Universal Fermi Interaction. We note also that this correction is present to the *same* extent in muon capture but absent in muon decay; the effective Fermi constant for the μ decay should, for this reason, be slightly different from its value for μ capture and N decay.

The last statement is rather surprising since earlier they observe: "one ought to add contributions from diagrams corresponding to wavefunction renormalization".

This brings us to the prescient observation of Gershtein and Zeldovich concerning the strong interaction effects on a V hadron current in the weak interaction. Yakov Zeldovich, who had been working on many fronts, was also concerning himself with the possible β-decay of the charged pion:[15]

$$\pi^- \to \pi^0 + e^- + \nu.$$

This was followed by the paper of Gershtein and Zeldovich[16] in which they critically re-examined the problems posed by Finkelstein and Moszkowski on the basis of covariant perturbation theory, and including the effect of nuclear wavefunction renormalization. They basically agreed with Finkelstein and Moszkowski, acknowledging that the covariant method is not really superior to the static calculation.

But the most important contribution in this paper[16] was a cursory remark that was hastily and wistfully dismissed! We quote the authors:

> It is of no practical significance but only of theoretical interest that in the case of the vector interaction type V, we should expect the equality [g' refers to bare coupling constant]:
>
> $$g_{F(\nu)} \equiv g'_{F(\nu)} \tag{4}$$
>
> to any order of the meson–nucleon coupling constant, taking nucleon recoil into account and allowing also for interaction of the nucleon with the electromagnetic field, etc. This result might be foreseen by analogy with Ward's identity for the interaction of a charged particle with the electromagnetic field; in this case, virtual processes involving particles (self-energy and vertex parts) do not lead to charge renormalization of the particle.

Gershtein and Zeldovich considered their idea of a conserved vector current (CVC) of "no practical significance" because they accepted the conclusion in a 1954 paper,[17] reporting on a measurement of the angular correlation in Ne^{19} – a mixed Fermi–Gamow–Teller transition – that the β interaction was a combination of S and T. This conclusion only followed if one believed in the He^6 result, an error which Gershtein and Zeldovich shared with many others.

The situation regarding the Lorentz structure of the β and other weak interactions changed rapidly with the discovery of parity violation in Co^{60} decay. The backward electron asymmetry found[18] in polarized Co^{60} gave unequivocal evidence for parity violation and could be explained (since the decay of Co^{60} was a Gamow–Teller transition) by a T interaction with a right-handed neutrino (ν_R) or an A interaction with a left-handed neutrino (ν_L). On the other hand, the backward electron asymmetry (with respect to the muon momentum) measured in muon decay[19] required the V (A) interaction for muon decay.

It appeared that UFI was in great trouble, and that the heroic surge of parity-violating experiments had only deepened the confusion – because if the β interaction was a combination of V and T, one would be forced to assign opposite helicities to the neutrinos emitted in Fermi (ν_L) and Gamow–Teller (ν_R)-type transitions, a most displeasing prospect! With this state of confusion prevailing vis-à-vis the parity-violating weak interaction experiments at the time of the 7th Rochester Conference,[20] the present authors withheld a report to that conference on their universal V–A theory until there was further clarification in the experimental situation. Without going into details here,[21] let us just say that we *were* convinced – at a crucial meeting with Felix Boehm at Caltech in early July (1957) – that all parity–violating experiments were consistent with V, A and ν_L *or* S, T and ν_R. With this assurance we were able to complete our paper within a matter of days, and to send off an abstract to the organizers of the Padua–Venice Conference where we expected to present our work in September 1957.[22]

Unfortunately, we did not know of the conserved vector current hypothesis of Gershtein and Zeldovich, or we would have taken the elegance of the CVC hypothesis as yet another argument for the universal V–A theory of weak interactions that we were proposing in the Padua–Venice paper. Instead, in that paper we made a comprehensive reanalysis of all the experimental data on parity-conserving and parity-violating experiments in β and muon decay, as well as the experiment on the $\pi \rightarrow e\nu/\pi \rightarrow \mu\nu$ ratio $\mathbb{R}$, and concluded that the only possible universal weak interaction was V–A with ν_L *and* that the universal V–A theory could survive only if the claims of four experiments at that time were mistaken. Those experiments were: (a) electron–neutrino angular correlation[7] in He;[6] (b) sign of the electron polarization from muon decay; (c) frequency of the electron mode in pion decay; (d) asymmetry from polarized neutron decay. Within the next 2 years – by 1959 – those four experiments were all redone and the new results were in complete accord with the V–A theory.[22] Within the same period Goldhaber and his collaborators[23] carried out their ingenious experiment to directly measure the neutrino helicity, which they found to be left-handed, thereby giving overwhelming support to the universal V–A theory.

In addition to the phenomenological arguments presented in our Padua–Venice paper in favour of the universal V–A theory, we did appeal to a new theoretical principle, that of chirality invariance, which, we argued, introduced an extra theoretical elegance into the V–A interaction. It is worth quoting this part of our original V–A paper to highlight the difference between the CVC hypothesis and chirality conservation. The additional remarks were as follows:

> One can rewrite the interaction of the four [Dirac] fields A, B, C, D, in the form:
>
> $$g\,\bar{\mathrm{A}}'\,\gamma_\mu\,\mathrm{B}'\,\bar{\mathrm{C}}'\,\gamma_\mu\,\mathrm{D}'$$
>
> where A′, B′, C′, D′ are the "two-component" fields: $\mathrm{A}' = (1/\sqrt{2})\,(1+\gamma_5)\mathrm{A}$, $\bar{\mathrm{A}}' = (1/\sqrt{2})\,\bar{\mathrm{A}}\,(1-\gamma_5)$, etc. Now the "two-component" field $(1/\sqrt{2})\,(1\pm\gamma_5)$ A is an eigenstate of the chirality operator with eigenvalues ± 1. Thus the universal Fermi interaction, while not preserving parity, preserves chirality and the maximal violation of parity is brought about by the requirement of chirality invariance. This is an *elegant formal principle*, which can now replace the Lee-Yang requirement of a two-component neutrino field coupling. . . . Thus our scheme of Fermi interactions is such that if one switches off all mesonic interactions, the gauge-invariant electromagnetic interactions (with Pauli couplings omitted) and Fermi couplings retain chirality as a good quantum number.

Underlying the above statement was our recognition that chirality (γ_5) invariance of the *charged* fermion current leads to the V–A Lorentz structure $\bar{\mathrm{A}}'\gamma_\mu\mathrm{B}'$ whereas the chirality invariance of a neutral fermion current (like the electromagnetic current) is consistent with a purely V parity-conserving Lorentz structure. It is the possibility of maintaining chirality conservation simultaneously for both charged parity-violating and neutral parity-conserving fermion currents that was exploited later in the breaking of the gauged chiral electroweak group $\mathrm{SU(2)_I} \times \mathrm{U(1)_Y}$ to the gauged non-chiral electromagnetic group $\mathrm{U(1)_{EM}}$.

The Padua–Venice formulation of the parity-violating universal Fermi interaction contained the V hadron current as a co-equal part of the total V–A interaction (maximal parity violation!) and it was now eminently sensible to work out the full implications of the Gershtein–Zeldovich idea. This was done by Feynman and Gell-Mann[24] in a paper written within months of our Padua–Venice paper; in this paper, they state:

> It might be asked why this agreement [between the muon and vector (nucleon) coupling constants] should be so good. Because nucleons can emit virtual pions, there might be expected to be a renormalization of the effective coupling constant. On the other hand, if there is some truth in the idea of an interaction with a universal constant strength, it may be that the other interactions are so arranged as not to destroy this constant. We have an example in electrodynamics. Here the coupling constant e to the electromagnetic field is the same for all particles coupled. Yet the virtual mesons do not disturb the value of this coupling constant. . . . The term $\frac{1}{2}\bar{\psi}\gamma_\mu\psi$ is conserved, but the term $\psi\gamma_\mu\tau_Z\psi$ is not conserved unless we add the current of pions $i[\varphi^* T_Z \nabla_\mu \varphi - \nabla_\mu \varphi^* T_Z \varphi]$, because the pions are charged. Likewise $\psi\gamma_\mu\tau_+\psi$ is not conserved but the sum
>
> $$J_\mu^V = \bar{\psi}\gamma_\mu\tau_+\psi + i[\varphi^* T_+ \nabla_\mu \varphi - (\nabla_\mu \varphi^*) T_+ \varphi] \tag{5}$$
>
> is conserved, and, like electricity, leads to a quantity whose value (for low-energy

transitions) is unchanged by the interaction of pions and nucleons. . . . The existence of the extra term in (5) means that other weak processes must be predicted. . . . Thus $\pi^- \to \pi^0 + e + \bar{\nu}$ should have the same *ft*-value as O^{14}.

In essence, Feynman and Gell-Mann (and Gell-Mann in a subsequent paper[25]) expressed Gershtein–Zeldovich's CVC hypothesis in more precise and useful language: the non-renormalization consequence of CVC in electromagnetism can be taken over for the charged vector part of the hadronic weak current if the global neutral (electromagnetic) and weak (charged) vector hadron currents belong to the same representation of the strong isospin (I) group $SU(2)_I$ (which they do) and one accepts isospin invariance. All the interesting consequences of the CVC hypothesis in weak interactions then follow: (1) the non-renormalization of the weak vector coupling constant, i.e. $g_v(q^2=0)=1$ (q^2 is the four-momentum transfer), in complete agreement with Gershtein–Zeldovich's original conjecture. The measured value of $g_v(0)$ actually turned out to be somewhat smaller than 1 – about 2%. Attempts to explain the discrepancy on the basis of radiative electromagnetic corrections did not succeed. The discrepancy soon found its explanation within the framework of the Cabibbo hypothesis (see below) and, indeed, provided an independent determination of the Cabibbo angle; (2) a unique and confirmed prediction of the branching ratio for the weak decay process originally considered by Zeldovich,[15] namely: $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$. The CVC hypothesis fixes the value of the form factor in the $q^2=0$ limit and the theoretically predicted branching ratio is 1.04×10^{-8}, agreeing with the observed value within the experimental uncertainty of a few per cent; and (3) "weak magnetism", which is a most refined test of the common behaviour of the electromagnetic and weak vector hadron currents because the comparison is being made for finite q^2. Because the electromagnetic and weak vector hadron currents belong to the same representation of $SU(2)_I$, the weak vector form factors can be expressed as linear combinations of the neutron and proton electromagnetic form factors, which are known experimentally. The nuclear isospin triad B^{12}, C^{12*} (15.11 MeV) and N^{12} is most suitable for testing "weak magnetism" because of the large available energies for the β transitions in B^{12} and N^{12}, as well as the γ transition from C^{12*}. Using CVC, the correction factor to the allowed β spectra (from B^{12} and N^{12}) can be related to the rate of the γ transition from C^{12*} and the test passes with flying colours.[26]

The confirmation of the Gershtein–Zeldovich–Feynman–Gell–Mann CVC hypothesis in weak interactions played an important role in helping to pin down the significance of the Cabibbo angle[27] – that was introduced in 1963 – to explain the suppression of (strong) hypercharge-violating ($\Delta Y=1$) decays (by a factor of about 20) compared to the (strong) hypercharge-conserving ($\Delta Y=0$) decays. The determination that $g_v(0)=1$ made it possible to obtain an independent value of the Cabibbo angle from the relation between the measured $\bar{g}_v(0)$ in $\Delta Y=0$ semi-leptonic decays and g_μ, the value derived from muon decay (i.e. from $\bar{g}_v(0)=\cos\theta\, g_\mu$, where θ is the Cabibbo angle). The measurement of the suppression factor of the $\Delta Y=1$ compared to the $\Delta Y=0$ semi-leptonic decays depends on $\sin\theta$, and both methods agreed on the value $\theta=13.4°$, lending support to the Cabibbo hypothesis and, subsequently, to the Kobayashi–Maskawa mass matrix approach[28] (with its three "Cabibbo angles" and its one CP phase) to explain the relation between the mass and weak interaction eigenstates of the three generations of quarks and leptons.

The CVC hypothesis had nothing to say about the axial vector hadron current, even for $\Delta Y=0$, and even though maximal parity violation required an equal admixture of vector and axial vector unrenormalized hadron currents. Since there was no strong renormalization effects for the lepton currents, the calculation of the axial vector current contributions for the leptons was as straightforward as that of the vector current contributions (as long as the weak currents were not gauged – see below). Prior to the gauging of the weak interaction, the CVC analogy was used to search for a "partially conserved" axial vector hadron current since it was clear that a "fully" conserved axial vector hadron current would forbid pion decay. So one looked for a conservation law violated to the extent that the pion has a non-zero decay amplitude (resulting from a non-zero mass). This is the hypothesis of the partially conserved axial vector current (PCAC):[29]

$$\partial^\mu A^a_\mu = f_\pi m_\pi^2 \phi^a \quad (a=1,2,3 \text{ for the SU(2)}_\text{I} \text{ group}) \tag{6}$$

where f_π is the pion decay constant measured for $\pi\to\mu+\nu$. This relation, together with the current algebra relation:[30]

$$[A_0^-(\mathbf{x},t), A_0^+(\mathbf{v},t)] = 2iV_0^3(x)\delta(\mathbf{x}-\mathbf{y}) \tag{7}$$

enabled Adler and Weisberger[31] to calculate the renormalized axial vector coupling constant from (extrapolated) pion–nucleon scattering data. The Adler–Weisberger relation for the axial vector renormalization may be thought of as the logical culmination to the efforts of Finkelstein and Moszkowski[31] and of Gershtein and Zeldovich.[16] Using the most recent values of the experimental parameters in the Adler–Weisberger relation, one obtains $g_A(0)=1.26$, a value quite consistent with the crude predictions of the earlier authors.

The Standard Model is built on the V–A interaction unifying it further with electromagnetism to produce an electroweak model. This model is renormalizable; and the radiative corrections to the model can be calculated and are finite. Since the experimental measurement for superallowed Fermi transitions have acquired an accuracy of one part in a thousand[32] the computations of radiative corrections are appropriate. There is a sharp discrepancy between uncorrected values for the four light nuclei (^{14}O, ^{26}Al^{m}, ^{34}Cl, ^{38}K^{m} and ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co). Sirlin and Zucchini[33] have done such a calculation of leading radiative corrections $O(Z\alpha^2)$, making use of the theorems on cancellations of mass singularities for total decay rates. The net result of the various radiative correction calculations is presented in Table 2. The discrepancy between light and heavy nuclei has disappeared and the distribution of values gives much confidence in the conserved vector current hypothesis. Along with this is the Standard Model picture with

$$V_{ud}^2 = V_{ud}^2 + V_{us}^2 + V_{ub}^2$$

Sirlin and Zucchini find

$$V_{ud} = 0.9747 \pm 0.0011$$

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9984 \pm 0.0023$$

The corrections are $\sim 4\%$, and if they were not included the unitarity bound of 1 would have been exceeded by $\sim 3.7\%$.

We have learned a good deal about the meaning of the PCAC hypothesis since it was first proposed more than two decades ago. We now understand from the quantum chromodynamics of the first generation of quarks (u and d) how the $u\bar{u}$, $d\bar{d}$ and $u\bar{d}$ quark condensates spontaneously break the global chiral quark flavour symmetry $SU(2)_L \times SU(2)_R$ down to the global flavour vector $SU(2)_I$ symmetry and produce the isotriplet of Goldstone pions in the process. The finite masses of the pions, i.e. their quasi-Goldstone particle status, results from the small but finite masses of the u and d quarks. It is the "diagonal" (vector) sum of $SU(2)_L$ and $SU(2)_R$ that the quark condensates leave conserved; it is the difference of the $SU(2)_L$ and $SU(2)_R$, namely the axial vector $SU(2)$ group, that is *not* conserved and whose breaking gives rise to the Goldstone pion states. This striking difference is encapsulated in the properties of the chirality operator γ_5, which defines the difference between a vector and an axial vector current. Indeed, it was the insistence on the chirality (γ_5) invariance of the charged weak fermion current that led to the V–A theory.[22]

TABLE 2. *Radiative corrections applied to β-decay Ft value*$_{32}$

Decay	Modified values
^{14}O	3073.4 ± 3.9
^{26}Al^{m}	3066.9 ± 3.9
^{34}Cl	3066.9 ± 5.0
^{38}K^{m}	3064.2 ± 5.1
^{42}Sc	3074.7 ± 7.9
^{46}V	3071.3 ± 5.2
^{50}Mn	3066.2 ± 6.5
^{54}Co	3068.6 ± 1.8

The idea of insisting on the γ_5 invariance of the charged weak fermion current was a generalization of the concept of a two-component neutrino,[32] and not only produced the correct Lorentz structure of the weak currents with their maximal parity violation, but subsequently opened the door – through chirality conservation – to the joining of the electromagnetic and weak interactions into the gauged chiral electroweak group $SU(2)_L \times U(1)_Y$ where $SU(2)_L$ is the (chiral) *weak* isospin group and $U(1)_y$ is the (chiral) *weak* hypercharge group. The surprisingly crucial role played by the γ_5 operator in modern particle physics only became apparent later – after the discovery of the axial ABJ anomaly[33] and the gauging of the chiral quark and lepton flavours into the electroweak group. We comment on this fascinating development in the briefest terms in order to place the banner years 1955–57 – commencing with the CVC hypothesis of Gershtein and Zeldovich and ending with the universal V–A theory – in proper historical perspective.

As is well known, the PCAC hypothesis became the starting point of the famous ABJ calculation of the decay $\pi^0 \rightarrow 2\gamma$ that – in quark language – acquired the form:

$$\partial_\mu j^\mu_A = 2m_q j_P + \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{8}$$

where j^μ_A is the axial vector quark current defined by:

$$j^\mu_A = \bar{q}\gamma^\mu\gamma_5 q. \tag{8a}$$

and j_P is the pseudoscalar quark density defined by:

$$j_P = i\bar{q}\gamma_5 q. \tag{8b}$$

In eq. (8), α = the fine structure constant, m_q = the quark mass, $F_{\mu\nu}$ = the electromagnetic field tensor and $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ the dual electromagnetic tensor. It is seen from eq. (8) that the global axial vector current is not conserved in the limit of massless quarks, and that the only way to maintain electromagnetic gauge invariance for the fermion triangle contribution to $\pi^0 \rightarrow 2\gamma$, is to introduce an "anomalous" term containing the electromagnetic field tensor and its dual. Indeed, it turns out that – in the soft pion limit – it is the anomalous term that provides the exclusive and correct contribution to the transition amplitude for $\pi^0 \rightarrow 2\gamma$ decay. Since the axial vector current in eq. (8) is a global current, the presence of the anomaly is permitted and, as we have just said, it is responsible by itself for the decay. The situation changes drastically when the axial vector current is part of a (chiral) gauge group as it is in the case of the electroweak group. In that case the axial vector current (of the massless fermions) is coupled to the weak gauge bosons of the electroweak group and the anomalous terms lead to the failure of the Ward identities and the lack of renormalizability of the theory for both quarks and leptons; the triangular (perturbative) anomaly must be cancelled. Fortunately, the quarks and leptons of each generation possess the right quantum numbers under $SU(2)_L \times U(1)_Y$ so that all the triangular chiral gauge anomalies cancel and the renormalizability of the electroweak theory is maintained. The cancellation of the triangular chiral gauge anomaly has turned out to be an important methodological tool for constructing unified theories of any type, since the larger groups must contain the chiral electroweak group as a subgroup.

It is of great interest to note that two other types of chirality-related anomalies in four dimensions have been identified, whose absence is required for the self-consistency of the electroweak theory. A second anomaly that arises in chiral gauge theories is the global (non-perturbative) anomaly, known as the Witten SU(2) anomaly. Witten[36] showed in 1982 that any SU(2) gauge theory with an *odd* number of left-handed fermion (Weyl) doublets (and no other representations) is mathematically inconsistent. There is no problem with an odd number of Dirac doublets – since each Dirac doublet is equivalent to two Weyl doublets. Mathematically, Witten showed that the fermion path integral (taken over the Weyl fermions) for an SU(2) gauge theory with an odd number of (massless) Weyl fermion doublets, changes sign (the change of sign is due to the properties of the chirality operator γ_5) under a topologically non-trivial SU(2) gauge transformation. This introduces ambiguities in the evaluation of expectation values of the quantum field operators, and leads to a mathematically inconsistent theory. The only solution is to insist on an even number of SU(2) Weyl doublets in a viable theory.[34]

The third type of anomaly discovered in four dimensions is the "mixed chiral–gravitational anomaly" that is similar to the usual perturbative fermion triangle anomaly in chiral gauge theories but with the three chiral current vertices replaced by a mixture of one chiral current vertex and two energy-momentum tensor (gravitational) vertices. This anomaly was first pointed out by Delbourgo and Salam;[37] its consequences were discussed by Alvarez-Gaumé and Witten[38] in 1983, who concluded that a necessary condition for consistency of the standard model coupled to gravity is that the sum of the hypercharges of the left-handed fermions vanishes, i.e. Tr $Y=0$. This anomaly-free condition is, fortunately, also obeyed by the standard electroweak theory, so that there is no "mixed chiral–gravitational" anomaly. Thus, all three anomalies in four dimensions – due, in some fashion, to the properties of the γ_5 operator – are absent for the standard chiral gauge electroweak group and, hence, electroweak theory should be completely renormalizable and self-consistent.

The further intriguing remark is that a GUT group that is free of the usual triangular chiral gauge anomalies, will automatically be free of the global SU(2) (Witten) anomaly and the "mixed–gravitational" anomaly;

the statement about the Witten SU(2) anomaly has recently been proved[37] and that about the mixed anomaly is obvious because, for a (simple) GUT group, Tr $Y=0$ holds because Y is a (traceless) generator of a simple group. We emphasize that, in principle, all three types of anomalies cited above are not present in vector-like (non-chiral) gauge theories, and arise in chiral gauge theories despite global chirality invariance. Since the future theory of everything (TOE!) will have to contain the standard chiral gauge group, the cancellation of chiral gauge anomalies of whatever sort – whether in four dimensions or in a higher number of dimensions (as in superstring theory) – must be achieved, and the resulting anomaly-free conditions will provide, and continue to provide, a powerful tool for the construction of TOE or any intermediate version of it!

We have come a long way in the development of weak interaction theory since the exciting few years of the mid-1950s. It started in 1955 with the conserved vector current hypothesis of Gershtein–Zeldovich,[16] continued in 1956 with the Lee–Yang paper on parity violation[40] and culminated in 1957 in the universal V–A theory of weak interactions based on the concept of chirality invariance.[22] Feynman and Gell-Mann had the good fortune of combining in their famous paper[24] the twin concepts of a conserved charged vector hadron current and the universal charged chiral V–A (hadron plus lepton) current. It is evident from the history of the past 30 years that this marriage – when combined with the concepts of Yang–Mills gauge fields[41] and spontaneous symmetry breaking[42] – hastened the arrival of the remarkably successful chiral gauge electroweak theory of Glashow, Salam and Weinberg[43] and its present-day ramifications.

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