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THE QUANTUM ENVELOPE OF A CLASSICAL SYSTEM[1]

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Physical quantities and the processes that change them are treated on an unequal footing in classical dynamics. Quantum theory identifies the two categories. It is shown how a classical system may be viewed as a quantum system with hidden variables by such an identification. Two applications are outlined. A quantum theory of measurement in which the apparati are classical, with few degrees of freedom, interacting with a quantum system. The mind-brain interface is amenable to a similar treatment: the beginnings of a quantum theory of consciousness are sketched. The two problems are not unrelated. Systems with discretely many states are not excluded.

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Introduction

Classical dynamical systems are generally taken to be suitable limiting forms of a larger quantum system with too many irrelevant degrees of freedom. Quantum mechanics is taken to be of universal validity; all physical objects including laboratory measuring apparati with only a single pointer are considered as the aggregate of a very large condensed matter system with most of its degrees of freedom frozen out and a few excited to the correspondence principle regime. The sole purpose of classical mechanics, in this widely held view, is as an approximation with limited validity[1]. The manifestation of macroscopic quantum effects like superfluidity and (low temperature) superconductivity lend support to this view. In exploring high temperature superconductivity we restore the microscopic quantum theory. Even in measurement theory where this picture is inappropriate, people try to appeal to laws of large numbers to guarantee the classical behavior of the measuring apparatus, notwithstanding the fact that an ideal apparatus should be a pointer governed by classical dynamics.

In this paper in honor of Professor Susumu Okubo, I explore and explain an alternate point of view in which a classical system is seen as a restriction of a quantum system with "twice" the number of degrees of freedom, not arbitrarily many. The classical systems and their counterpart systems therefore may have only a few degrees of freedom, not 10^{23} degrees of freedom. The relationship is not one of averaging over large numbers nor the correspondence principle limit but by means of a (generalized) superselection principle. This associated quantum system is realized in a natural manner and is called "the quantum envelope of the classical system".

The appropriateness of these ideas in the context of quantum measurement theory was outlined a dozen years ago[2] and applied to certain specific cases of measurement[3]. But the general validity of this notion and the mathematical structure did not get adequately exposed; nor was the use of transformations by vector fields exhibited in its

generality. It is my hope that this presentation would remedy these defects. The application to a quantum theory of consciousness need not scare away the orthodox physicists.

The Quantum Envelope

Consider a classical dynamical system with $\nu/2$ canonical degrees of freedom with the ν fundamental dynamical variables q_i, p_i and Poisson bracket

$$[g(q,p), f(q,p)] = -\frac{\partial f}{\partial q}\frac{\partial g}{\partial p} + \frac{\partial f}{\partial p}\frac{\partial g}{\partial q} = (F \cdot g)(q,p)$$

, where F is the vector field

$$F = \frac{\partial f}{\partial q}\frac{\partial}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial}{\partial q}\,.$$

The correspondence of dynamical variables to vector fields is many-to-one since both f and f + a, where a is any constant, have the same vector field.

The transformation generated by a dynamical variable is realized by the corresponding vector field[4]. In particular if h(q,p) is the Hamiltonian function and

$$H = \frac{\partial h}{\partial p}\frac{\partial}{\partial q} - \frac{\partial h}{\partial q}\frac{\partial}{\partial p}$$

is the corresponding vector field, the equations of motion are

$$\frac{d}{dt}g(q,p) = (H \cdot g)(q,p).$$

Equally well, if we consider translations or rotations the corresponding generator and vector fields are:

$$p_j \cdot P_j = \frac{\partial}{\partial q_j}: \qquad translations; |\geq j \leq 3;$$

$$\epsilon_{ijk} q_j p_k \rightarrow J_j = \epsilon_{ijk} q_i \frac{\partial}{\partial q_k}: \qquad rotations; 1 \leq j,k \leq 3.$$

For the classical system all dynamical variables are observable, but none of the vector fields are observable. The dynamical variables corresponding to any of these

vector fields (which are defined only up to an additive constant) are certainly observable. For example the energy function $h(q,p)$ is certainly observable though the Hamiltonian vector field H is not an observable. We also recognize that $h(q,p)$ is a constant of motion:

$$\frac{d}{dt}h(q,p) = (H \cdot h)(q,p) = 0 \ .$$

More generally the momentum, angular momentum $\ell = \mathbf{q} \times \mathbf{p}$ momentum $\mathbf{p}$ energy $kp^2/2m = h$ and the mechanical moment $mq = \mathbf{g}$ for a nonrelativistic particle obey the usual transformation laws:

$$\begin{aligned} P_j p_k &= 0; \quad P_j(q \times p)_k = -\epsilon_{jk\ell} p_\ell; \\ P_j h &= 0; \quad P_j g_k = -m\delta_{jk}. \end{aligned}$$

$$G_j h = p_J \ \ G_j g_k = 0 \ \text{ etc.}$$

These (and the rest of such equations) express the behavior of the physical quantities as the frame transformations of the Galilean type are (infinitesimally) implemented. While the mechanical moment changes under translations and the change is proportional to the free particle mass, the corresponding vector fields satisfy the Lie algebra of the Galilei group. In particular,

$$G_j P_k - P_k G_J = 0.$$

Corresponding relations obtain for the commutator bracket of the boost $\mathbf{G}$ and the time translation H.

Since only the primary canonical variables q, p are observable we may (formally) consider (ideal) eigenstates of these states and the wavefunction $\psi(q,p)$ in the Schrödinger representation. Since they belong to the continuous spectrum ψ is not normalizable:

$$\psi(q,p) = \delta(q-q')\delta(p-p') \ e^{i\phi(q',p')}$$

This corresponds to a classical particle at the phase space point (q', p'). A normalizable state is obtained by taking superpositions of such states:

$$\psi(q,p) = \int\int \varphi(q',p')\delta(q-q')\delta(p-p')e^{i\phi(q'p')}dq'dp' = \varphi(q,p)e^{i\phi(q,p)}.$$

This corresponds to a classical particle with a phase space density $\rho(q,p)$. Since any operator which has matrix elements which connect different values of q and p contain exponentials of vector fields they are unobservable. Hence the phase $\phi(q,p)$ is entirely arbitrary. So for all practical purposes only

$$\rho(q,p) = \mid \psi(q,p) \mid^2$$

is measurable[5]. However, it is ψ or $\mid \psi \mid$ which is relevant as the wavefunction, *not* ρ ! The states corresponding to distinct values of q,p are superselected .

I must call attention to the fact that the superselection principle is distinctly different from the superselection principle used in quantum theories with superselection sectors[6]. Here we can pass from one state to another state by virtue of the time evolution (or other transformations) since the concerned generator is not an observable.

More generally let us consider a classical system with ν variables ω_μ, $1 \leq \mu \leq \nu$, with equations of motion

$$\frac{d}{df}f(\omega) = (H \cdot f)(\omega)$$

with H a suitable vector field on $f(\omega)$. Other transformations are brought about by other vector fields. The vector fields form a Lie algebra under commutation:

$$\begin{aligned} F &= f^\mu(\omega)\frac{\partial}{\partial\omega\mu}, \quad G = g^\mu(\omega)\frac{\partial}{\partial\omega\mu} \\ FG - GH &= (F,G) = f^\mu_{(\omega)}\frac{\partial g^\nu}{\partial\omega^\mu}\frac{\partial}{\partial\omega^\nu} - g^\mu(\omega)\frac{\partial f^\nu_(\omega)}{\partial\omega^\mu}\frac{\partial}{\partial\omega^\nu} \ . \end{aligned}$$

When the classical system admits a symmetry group, the symmetry is realized by the vector fields associated with the corresponding Lie algebra. In addition to such continuous

group elements there may also be discrete elements which are realized by the exponentials of suitable vector fields or by other integral operators.

Since the Hamiltonian operator H is a vector field over the classical phase space, the quantum equations of motion of (q, p) are identical with the classical equations of motion.

Most of the comments for the canonical classical system can be carried over to this generalized framework. The states are realized by wavefunctions $\psi(\omega)$ with $\psi(\omega)$ and $\psi(\omega)e^{i\phi(\omega)}$ being physically equivalent.

It is known that in both classical dynamics and quantum dynamics the free particle obtains as an elementary (irreducible) realization. The same applies in the present case also except for the gauge freedom

$$\psi \rightarrow \psi e^{i\theta}.$$

More generally we can ask for a complete set of vector fields which have the property that the only invariants of the set of vector fields are multiples of the identity. It is easy to verify that if the original classical dynamical system was irreducibly realized the vector fields are complete. The gauge freedom corresponds to the possibility of extending the vector fields with nongradient terms.

Given the classical system with $\nu/2$ degrees of freedom and its quantum envelope with ν degrees of freedom we may recognize that the vector fields themselves under ordinary associative multiplication (i.e., the constant vector fields) constitute a quantum system with $\nu/2$ degrees of freedom. It is entirely unobservable with the superselection principle we imposed. But in – and of – itself, these constant vector fields under associative multiplication, constitute a counterpart to the primary classical system. The quantum envelope may be viewed as a projective realization of this "disembodied system". For ν degrees of freedom $\frac{\partial}{\partial q}$, $\frac{\partial}{\partial p}$ form a commutative system. Yet its projective realization[7] in terms of function of q and p constitutes a quantum system with ν degrees

of freedom. The use of the projective realization may be viewed as quantization.

Application to Measurement Theory

When a quantum variable is to be measured we have to couple it to an apparatus which is to be viewed as classical. The (microscopic) quantum system should affect the (macroscopic) classical apparatus to manifest the measurement. The apparatus should, in turn, affect the quantum system and assure that it is left in an eigenstate of the commuting set of variables measured. We need therefore an *interaction* between the *classical* apparatus and the *quantum* system. We don't need a quantum apparatus with too many degrees of freedom but a *classical* apparatus. I believe the first such attempt has been a model I presented a few years ago[2].

The composite of the quantum system and classical apparatus has a noncommutative set of dynamical variables; and hence, should be treated quantum mechanically. To do this we take the quantum envelope of the classical apparatus with its restriction to observing only the primary variables. When we put this quantum envelope in interaction with the quantum system subject to the measurement we must continue to require that the previously observables continue to be observables under time development and that there are no more variables of the apparatus that are observable created by the coupling. This leads to the restriction that at most only a complete set of commuting observables can be measured[2].

Let the classical system have phase space variables ω. We introduce the superselected quantum envelope by adding the gradients $\frac{\partial}{\partial\omega}$: for convenience we denote by π the conjugate momenta so that

$$\pi_\nu = -i\frac{\partial}{\partial\omega^\nu}.$$

The Hamiltonian operator for this system is the vector field

$$H_2^o = \frac{\partial h}{\partial\omega^\nu}\epsilon^{\mu\nu}\cdot i\pi_\nu.$$

For the measured quantum system the Hamiltonian operator is

$$H_1^o = X(\xi),$$

where ξ are its dynamical variables and X is a suitable function. The interaction is chosen to be

$$H_3 = \phi^\mu(\omega,\xi)\pi_\mu + X(\omega,\xi)$$

where $X(\omega,\xi)$ includes $X(\xi)$ and any interaction terms solely dependent on ω. Then the equations of motion can be written

$$\begin{aligned} \frac{d}{dt}\omega^\mu &= i\phi^\mu(\omega,\xi); \\ \frac{d}{dt}\xi &= i[\xi,\phi^\mu(\omega,\xi)]\pi_\mu + i[\xi, X(\omega,\xi)]. \end{aligned}$$

In turn, we find that the apparatus variables ω^μ are not dependent on the quantum variables ξ. To guarantee that the apparatus variables commute amongst themselves we must restrict $\phi^\mu(\omega,\xi)$ to depend only on a commuting set of ξ. In turn, this requires

$$[\phi^\mu, X(\omega,\xi)] = f^\mu(\phi(\omega,\xi),\omega).$$

These are expected restrictions: we anticipate measurement of only a commuting set of quantum variables.

A simple example is provided by the measurement of spin by the splitting of a molecular beam in the Stern-Gerlach experiment[2,3]. Here the spin **S** is the quantum system of which the third component S_3 is to be measured. The translation degrees of freedom **q**, **p** of the molecular beam is treated classically with a well defined trajectory for the beam. The Stern-Gerlach Hamiltonian is

$$H = -\frac{i}{m}\mathbf{p}\cdot\frac{\partial}{\partial\mathbf{q}} - i\Gamma S_3\frac{\partial}{\partial p_3} - \gamma B S_3,$$

where B is the magnetic field along the third director, γ the gyromagnetic ratio and Γ is proportional to the magnetic field gradient. If the initial momentum is along the first direction with value p_1 the trajectory is given by

$$\begin{aligned} q_1(t) &= \frac{1}{m} t p_1 \quad q_2(t) = 0 \\ q_3(t) &= \pm \frac{1}{2} t^2 \Gamma \end{aligned}$$

where $\pm$ corresponds to the spinvalues $\pm\frac{1}{2}$. We get two parabolas.

As long as they are spatially separated the two components of the wave function have phases that are gauge dependent and unobservable. They have phases which are available but not availed of in the experimental set up. If by a suitable choice of a further Stern-Gerlach field etc. we bring the two parabolas together we would get superposition of the two component wavefunctions; the phase at the stage of superposition is no longer gauge dependent; it has observable consequences. A more systematic discussion of the Stern-Gerlach system is available in two papers which I coauthored with Sherry and Gautam[3].

For another example let us take a classical harmonic oscillator

$$h = \frac{1}{2}(m^{-1}p^2 + kq^2).$$

The corresponding quantum Hamiltonian is

$$H = i(kq\frac{\partial}{\partial p} - m^{-1}p\frac{\partial}{\partial q})$$

which leads to anticipated equations of motion:

$$\dot{q} = m^{-1}p, \dot{p} = -kq.$$

We can see this system with 2 quantum degrees of freedom:

$$q_1 = q, \; q_2 = p;$$

$$p_1 = -i\frac{\partial}{\partial q}, \quad p_2 = -i\frac{\partial}{\partial p},$$

from a different perspective by writing

$$\begin{aligned}
Q_1 &= \frac{1}{\sqrt{2}}(q_1 + p_2) = \frac{1}{\sqrt{2}}(q - i\frac{\partial}{\partial p}) \\
Q_2 &= \frac{1}{\sqrt{2}}(q_1 - p_2) = \frac{1}{\sqrt{2}}(q + i\frac{\partial}{\partial p}) \\
P_1 &= \frac{1}{\sqrt{2}}(p_1 - q_2) = \frac{-1}{\sqrt{2}}(p + i\frac{\partial}{\partial q}) \\
P_2 &= \frac{1}{\sqrt{2}}(p_1 + q_2) = \frac{-1}{\sqrt{2}}(-p + i\frac{\partial}{\partial q}) \, .
\end{aligned}$$

The quantum Hamiltonian now takes the form

$$H = 2\{(kQ_2^2 + m^{-1}P_2^2) - (kQ_1^2 + m^{-1}P_1^2)\}.$$

It is thus the direct sum of two oscillators, one with positive energy and one with negative "energy". The stationary states are discrete and the energies are $n\,\omega$ with n any integer, positive negative or zero. But for our purposes we are *not* interested in this basis functions or this representation. We want to have the wave functions in the "Schrödinger" representation with q and p diagonal; i.e. $Q_1 + Q_2$ and $P_1 - P_2$ diagonal. In this representation the point $q = p = 0$ is a stationary (infinitely degenerate) state of H and corresponds to zero energy h. No other phase point (q, p) is a stationary state; the stationary states (which are infinitely degenerate) appear as extended wavefunctions. This reproduces the fact that for the classical harmonic oscillator the only equilibrium point is (0, 0).

Systems with Finitely Many Extremal Configurations

A classical system whose extremal (pure) states are N discrete states necessarily

has only a stochastic dynamics. The generic (mixed) states are probability vectors

$$\{x_j\}: \quad x_j \geq 0; \quad \sum_{j=1}^{N} x_j = 1.$$

The stochastic dynamics of the probability vectors is by the vector equation

$$\frac{d}{dt} s_j = \sum M_{jk} x_k.$$

Then

$$m_{jk} \geq 0 \quad j \neq k; \quad \sum M_{jk} = 0.$$

The eigenvalues of the matrix M may be real or complex; except for one (or more) zero eigenvalues all other eigenvalues must have a negative real part. The complex roots occur in complex conjugate pairs.

We not have only discrete translations. They may be denoted by $T^{\alpha\beta}$ which translates the element β to the element α and annihilates everything else:

$$(T^{\alpha\beta})_{jk} = \delta_{\alpha j} \delta_{\beta k}.$$

Clearly the set $T^{1\beta}$, $T^{\alpha 1}$ constitute a generating set since

$$T^{\alpha\beta} = T^{\alpha 1}\, T^{1\beta}.$$

Any stochastic matrix can be expressed in terms of appropriate linear combinations of $T^{\alpha\beta}$. The $T^{\alpha\beta}$ are not observables but may be introduced as nonobservable dynamical variables. When so extended we get as the full set of dynamical variables the $N \times N$ matraices; and for generalized states the hermitian nonnegative density matrices with unit trace.

The system of dynamical variables in the set of all $N \times N$ matrices which is certainly noncommutative. Only the diagonal matrices are observables. The quantum envelope is the quantum mechanics of N level systems. Stochastic dynamics is the special case of

the evolution of N-level quantum systems in which diagonal density matrices continue to be diagonal. It is therefore a restriction of the dynamical maps of density matrices[8].

I have shown elsewhere[9] that all stochastic maps may be obtained by the contraction of the dynamics of an N^2 dimensional system undergoing generalized unitary evolutions; and that for completely positive maps the enlarged system has positive definite metric. The maps associated with classical linear stochastic dynamics can be obtained as a special case of these.

Quantum Envelopes and Automorphisms: Higher Envelopes

Given the quantum envelope often we can extend it by the (outer) automorphisms. For example the three vector classical translations have as quantum envelope the set of translations and coordinates (or boosts). This envelope has the rotations as automorphisms to get the Euclidean group[10]. This higher envelope can be further extended by reflections. In this manner we go to higher level envelopes with realizations built on the primitive (classical) configurations.

Having obtained the higher envelope we may seek other possible realizations built on different configurations.

Towards a Quantum Theory of Consciousness

It is of some interest to explore whether the ideas outlined here provide the beginnings of a quantum theory of consciousness. We know that, as far as our experience goes, brain function, mind and consciousness are all related. Brain functions, to a large extent, seem to be manifested through the on-off states (firing and quiescent states) of the multitude of neurons. The neurons function electrochemically: while electrochemistry has ultimately to be understood by quantum atomic chemistry, the essential features of neuronal functioning and networking seem to be describable classically. In most contemporary brain research[11] the descriptions are classical. Yet the functioning of the mind

does not find such a satisfactory description in classical terms. Notions of superposition, ambiguity, indecision, insight and creativity seem to have more quantum features. How could a brain adequately described by classical physics be related to a mind with its subtle quantum aspects?

A recent paper of Eccles[12] develops the notion of units of brain functioning (dendrons) and units of mental functioning (psychons) in interaction. Eccles identifies the minicolumn, "a vertically oriented cord of cells formed by the migration of neutrons from the gerominal epithelium of the neural tube along the radial glial cells to their destined locations in the cortex..." as the basic modular unit of the neocortex. "There are 80 to 100 apical dendrites in a dendron;... there are about 40 million such units in the human neocortex... this number of dendrons is adequate for providing the unitary basis for the neocortex in all of its extreme diversity and subtlety."

In a recent paper, Stapp[13] has outlined a quantum theory of consciousness. While I find gaps in his arguments he has analyzed the question of our awareness or "fell" in relation to brain processes. I quote: "Why, when we look at a triangle, do we experience three lives joined at three points, and not some pattern of neuron firings?

To answer this question let us consider first Edelman's explanation of how the visual cortex comes to be organized. The problem is this: the growth of the neurons connecting the retina to the visual cortex is not completely determined by genetic programming: there is a great deal of contingency. But then how does the structural information present at the retina get properly reconstitute at the cortex, rather than becoming hopelessly scrambled by the randomness of the neural connections.

The answer is that the saccadic movements of the eye cause the neurons that receive signals from adjacent retinal regions to receive temporally correlated signals. The resulting temporally correlated patterns of excitation in the visual cortex then become automatically associated, by the facilitation process. Thus some of the structure at the

retinal level becomes mapped into an *analog* structure in the realm of the cortical patterns of excitation.

Building up from this initial organization, initiated by the saccadic eye movements, repetitious patterns occurring at the retina *facilitate* corresponding patterns in the cortex.

Thus even though the neural wiring is haphazard, the process of facilitation nevertheless automatically established *analogs* of attended or recurring retinal patterns within the realm of the cortical patterns of excitations.

Patterns present in the visual cortex become associated, in the same way, with the neural accompaniments of those motor actions that bring them into being. Thus recurring features of the external visual scene will come to be associated with complex patterns of excitations that include the patterns that produce the motor actions that allow these features to be sensed.

Due to this mapping of structure the cortical patterns generated by attention to the external triangle will be "congruent" to the external triangle. For example, the adjacency properties of the points along the three lines of the triangle will have their symbolic representations among the cortical patterns originally facilitated by the saccadic eye movements. Similarly, the various other perceived structural features of the external triangle will be represented in symbols that have been previously automatically constructed by brain processes to represent these perceived connections. Thus focussing one's attention to the triangle will lead to an actual event that will select a "chord" of symbols, and this chord will be "congruent" to the external triangle, in the sense that it will contain symbols that are the analogs of the various structural features that characterize the external triangle itself. According to the theory what is felt, when one looks at the external triangle, is this chord of symbols, which is congruent to the external triangle.

It might seem that this shift from the external triangle to a congruent inner representation has not helped at all, but only made things worse: granting the congruency

property, still, Why do we experience the triangle rather than the firings of neurons?

This problem is the problem of symbols: How can one thing come to "represent" something else. If we posit an agent that simple sets up a correspondence then there is no problem. But we do not wish to introduce an outside agent that simply decrees that one thing shall be felt as something else, or a homnuculus that looks at the brain, and is able to decipher it and see a triangle.

This problem arises, however, only if one slides back to the classical concepts. The lesson of quantum theory is that there are no "things", but only acts: *there are no neuron firings*: there are only actual events. Thus the actualness, or feel, of an actual event must represent not "thing-ness", but "act-ness".

Symbols normally require an interpreter. This is the case also here: our symbols are interpreted by the unconscious processes. These symbols, when actualized by an actual event, *do things*: they act. Their actuality, or actualness, or feel, resides in what they do. One must not unwittingly revert to the classical ontology, and try to think of reality as existing primarily in the material particles. According to the quantum ontology those material particles are some sort of ephemeral or imaginary construct that, when cast into a suitably abstract mathematical form, can allow us to calculate statistical predictions about the actual things, which exist at a much higher level of integration, and are best understood as instructions, or intentions. It is among this sort of things that we must seek the actual, and hence the feel.

Starting, then from the feel, we can look for its counterpart in the abstract mathematical model. In the case of a human conscious act its counterpart must, according to this theory, lie in the realm of chords of symbols actualized by an actual event, and the feel of the chord resides in the *function* of the initiated act, i.e., in what it does. But what it does can be observed. In this way human consciousness can become naturally imbedded in the mathematical description of nature created by the physical sciences."

At this point I take leave of the theories of Eccles and of Stapp and take the conservative position that classically we have the brain and the brain processes. They are classical. We could construct the quantum envelope of this classical system. The functioning of this quantum envelope, of itself, is "awareness". This is the primary level of mind functioning. The "feel" of Stapp, the awareness, is perception of this envelope. It may be subject, by virtue of the superselection to only classical brain functioning; but that is not the way we feel. The brain feels pain and pleasure but not because it is classically in the brain. Already at the level of intelligent functioning we have risen from the classical system to its quantum envelope.

We do have higher mental functions: perspective on our mind, discrimination, creativity. These include transformation of mind function and is thus a higher envelope of the quantum envelope. It is not given to every creative writer or mathematician to explore his creativity but such instances are known[14]. Most people are unable to witness the transformation from waking to dreaming or to deep sleep states but that too is not impossible. In this way we may distinguish higher envelopes and be able to function at these levels in a manner that defies description at the lower levels.

Concluding Discussion

Kinematics already involves substance (configurations of objects, states of matter) and their changes. Dynamics is the kinematics of kinematics, the study of changes in uniform motion, changes in phase space. In other words dynamics involves substance and process. Classical dynamics distinguished dynamical variables and the processes in which they change. Dynamical variables are considered commuting physical quantities; but processes, though they may be generated by commuting physical quantities from a noncommutative algebra. In quantum theory processes and physical quantities are treated on the same footing. The quantum envelope is generated by the physical

quantities and the vector fields of physical processes of a primitive classical system.

The possibility of constructing a quantum envelope for any classical system allows the construction of the quantum envelope of a chaotic system like the kicked oscillator. These envelopes are examples of quantum chaos. There are, therefore, innumerably many chaotic quantum systems.

In this presentation I have discussed two possible applications of the concept of a quantum envelope; one to a precise description of quantum measurement process; and the other to outline a quantum theory of consciousness in which the brain processes are classical and the mind is in the quantum envelope. The higher mind functions involve changes in modality and in the nature of awareness: they thus involve a "remaking of the world" without changing the external classical world[15].

Classical dynamics may be viewed as a quantum theory with hidden (or forbidden!) variables.

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13. Henry P. Stapp, A Quantum Theory of Consciousness, report LBL-28574 Feb. 1990; presented at the First International Workshop.

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15. I am aware that I have given no quantitative treatment nor have I given measurement protocols.