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THEORY OF APPROXIMATE SYMMETRIES*

E.C.G. SUDARSHAN
DEPARTMENT OF PHYSICS,
SYRACUSE UNIVERSITY, SYRACUSE, N.Y.
UNITED STATES OF AMERICA

INTRODUCTION

In this paper I shall devote my attention to a discussion of several aspects of the theory of approximate symmetries of strong interactions from a unified point of view. The fundamental idea is that approximate symmetries are actually "masked symmetries", i.e. they are exact symmetries amongst states which are related to the natural (particle) states by a non-trivial unitary transformation. It is further postulated that these unitary transformations are directly related to the structure of strong interactions and are automorphisms of the Lie algebra of currents. The generators of the symmetry group are identified with the coupling vertices in physical processes like nuclear beta-decay. As an application of the method, a new sum rule for the renormalization of the Gamow-Teller coupling constant is derived.

In sections I and II we review the framework of approximate symmetries and exhibit its equivalence with the current algebra formulation. In section III we discuss some examples of the possible dynamical mechanisms generating symmetry and exhibit the existence of certain automorphisms of the self-consistent dynamics. These dynamical automorphisms are traced in detail by the use of models in section IV. Section V discusses the notion of masked symmetries. Section VI deals with the application to the derivation of the sum rule for the Gamow-Teller coupling constant. Some speculations about covariant symmetries are made in section VII.

I. THE GROUP-THEORETIC FRAMEWORK

In recent years it has become clear that strongly interacting particles, baryons and mesons, fall into multiplets and that these multiplet structures are intimately associated with their interaction properties. Going beyond the particle-antiparticle pairing for all elementary particles, one finds that the hadrons fall into isotopic multiplets which apparently organize themselves into unitary super-multiplets and into yet larger categories. It has been customary to interpret[1] this clustering of hadrons as a manifestation of an underlying group-theoretic structure with the particles constituting unitary representations of the relevant groups.

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[1] This interpretation is by no means inevitable. The advantage of contemplating beautiful structures which were not representations of groups was discussed by Gamba elsewhere in these Proceedings.

The earliest and most successful association of a group with multiplet structure was the identification of the neutron-proton doublet as a spinor with regard to the isotopic spin group and the brilliant prediction of the neutral pion and of the structure of pion-nucleon Yukawa interaction; and culminating in the identification of this structure as invariant under the isotopic spin group [1]. The strange particles and resonances (quasi-stationary scattering states) were then automatically organized in terms of isospin multiplets. These have then been cast into unitary multiplets with a corresponding postulate that their interactions are invariant under the unitary group in three variables. While the unitary multiplet classification is highly successful we are in no position to assess the quantitative success of the postulate of a unitary-invariant interaction structure.

Given a symmetry group for classifying the particles (as irreducible unitary representations) we could use it to classify dynamical variables in terms of (irreducible hermitian) tensor operators. Such a classification leads to immediate, experimentally verifiable (or refutable!) predictions. For example the identification of the electric charge-current density of hadrons as a linear combination of a scalar and the third-component of a vector operator with respect to the isospin group leads to the prediction that the magnetic moment (or any other linear electromagnetic property) is linear in the third component of the isospin and in particular to the well-known equality [2][2]

$$\mu(\Sigma^+)+\mu(\Sigma \quad = 2\mu(\Sigma^0).$$

For the same group the identification of the electromagnetic self-energies as a sum of three tensor operators leads to the mass difference formula

$$m(N_*^{++}) - m(N_*^{-}) = 3m(N_*^{+}) - 3m(N_*^{0}).$$

The electric charge-current density is further restricted by its identification as an octet tensor with regard to the unitary group; this gives further relations between the linear electromagnetic properties which have been extensively discussed in the literature. There is of course no special restriction to electromagnetic properties; in fact the best-known relation of this type following from the postulated tensor operator properties is the Gell-Mann-Okubo mass formula.

In deducing these consequences one does not need to assume that the group considered is an invariance group of the Hamiltonian since the predictions are automatic consequences of the postulated tensor characterization of the operators. This is particularly relevant in view of the pronounced lack of equality of the masses of particles within a multiplet. We might remind ourselves that the symmetry group of relativistic quantum mechanics, namely the Poincaré group (or its non-relativistic version, the Galilei-group), is believed to be an exact symmetry, though the Hamiltonian is by no means invariant under this group.

[2] This seems to be the first application of its kind to particle physics.

For example, in the SU(6) theory we would identify the components of the 56-component baryon multiplet and the 35-component meson multiplet suitably; and at the same time specify the transformation properties of, say, the magnetic moment operator. Though the Hamiltonian is not invariant under this group, there is nothing approximate about the structure of the representation or the "geometric" dependence of the matrix elements of tensor operators. From the assumption that the magnetic moment operator is the component of an adjoint tensor we obtain the now familiar $-2/3$ ratio between the neutron and the proton magnetic moments [3].

More generally, from the postulated unitary representation furnished by the physical states we have a set of hermitian ("current") operators with prescribed Lie algebra commutation relations. If we believe that the symmetry group commutes with, say, spatial momenta, the "current" operators will have non-vanishing matrix elements only between states of the same momenta. It is consistent to represent such translation invariant operators with space integrals of "local" current density operators, if we so choose. Thus, given the multiplets and the symmetry group we can define a "current algebra", without any entailed restrictions on the Hamiltonian.

II. EQUIVALENCE OF THE CURRENT ALGEBRA AND THE GROUP-THEORETIC FORMULATIONS

Several people [4] have in recent years sought to avoid temporarily the embarrassment of having to deal with approximate symmetries by trying to base the discussion on the algebra of "current" operators. The formulation involving the (space) integrated currents has been used recently to rederive several results of the SU(6) and SU(4) theories in a "more direct" fashion [5]. By the converse of the demonstration of the above paragraphs we can see that, insofar as one restricts attention to the integrated current operators, this is completely equivalent to the standard formulation [6][3].

For demonstrating this equivalence [7], we may start from the (integrated) current algebra and the physical identification of the dynamical variables. It is unnecessary to specialize to detailed assumptions about the structure of, say, the magnetic moment operator in terms of current densities or to the postulate of underlying canonical fields. In such cases we have a hermitian (finite-dimensional) representation of a (compact) Lie algebra of current operators. We can now define a unitary representation of the (local) Lie group by exponentiating the representation for the Lie algebra. In the case of a compact Lie algebra the hermitian representations are completely reducible, and the representations of the group are consequently reducible. (There may be dynamical information contained in the unitary transformations used for the reduction - see below.)

If we are given the current algebra and the multiplet we will, in general, still have ambiguities about the identification of their transformation properties. For example, the SU(4) current algebra acting on a multiplet containing

[3] Additional interrelationships of the reduced matrix elements can be traced by using the (postulated) commutation relations of the current density operators.

the nucleon can treat the nucleon as belonging to 4 or $\bar{4}$ or as part of 20 or $\overline{20}$, and so on. We could eliminate the first two alternatives, if we so choose, by normalizing the isotopic spin current matrix elements of the nucleon isobars. Even then the last two alternatives are equally good and give an ambiguous prediction for the ratio of the proton and neutron magnetic moments [8]. It is easy to show that this ambiguity is associated with the (outer) automorphism of the SU(4) current algebra generated by the extended charge conjugation operator. This ambiguity is equally present in the group-theoretic formulation. The existence of such automorphisms of the Lie algebra under study and the consequent change in the experimental predictions is much more general and will occupy us in the following sections.

From force of habit, at least, we would like the Hamiltonian to be either invariant or approximately invariant under the symmetry. In the case of isotopic spin symmetry this is more or less satisfied, and we believe that we can quantitatively account for the deviations. This is not so for unitary symmetry and its further developments. Several people have, in recent years, searched for a dynamical origin for symmetries and for the approximate nature of these symmetries.

III DYNAMICAL ORIGINS OF SYMMETRY

Consider several multiplets of interacting particles. Let us assume that the various members of any one multiplet have approximately equal masses. In course of interaction we expect that the physical masses of these particles would, in general, undergo varying changes. The assumption that the interaction between the particles is self-consistent would then lead to restrictions on the coupling between the particles. The earliest discussion along these lines [1] is almost as old as meson theory but the subject has been extensively studied in recent years [9]. The self-consistency arguments could be applied to propagators [10], vertices [11] or scattering amplitudes [12] or to several of them. We will discuss two special cases below chosen for their simplicity but rich enough to illustrate the relevant dynamical features.

For the first case consider "pion-nucleon" Yukawa interaction. We shall not assume electric charge conservation, but shall assume baryon number conservation. The coupling vertex can be written

$$\sum_{\alpha, r, s} \{g^{\alpha}_{rs} N^{\dagger}_r N_s \pi^{\alpha} + (g^{\alpha}_{rs})^* N^{\dagger}_s N_r \pi^{\dagger\alpha}\}.$$

Without any loss of generality we could assume that the pion field components are hermitian. It then follows that the three matrices g^{α} with matrix elements g^{α}_{rs} are hermitian. If we consider each particle to be a bound state of the two particles to which it is coupled (i.e. the nucleon as a bound state of the nucleon and pion, the pion as a bound state of the nucleon and the antinucleon) we could identify the composite particle wave functions to be proportional to g^{α}_{rs} multiplied by a suitable spatial wave function independent of the indices. The normalization conditions on the wave functions can then be written:

$$\sum_{rs} g^{\alpha}_{rs}(g^{\beta}_{rs}) \equiv \mathrm{Tr}\,(g^{\alpha}g^{\beta}) = A_1\delta^{\alpha\beta},$$

$$\sum_{\alpha s} g^{\alpha}_{rs}(g^{\alpha}_{ts})^{*} \equiv \sum_{\alpha} (g^{\alpha}g^{\alpha})_{rs} = A_2(\delta_{\gamma_5})_{rs}$$

The constants A_1, A_2 are not independent but are related by

$$2A_2 = 3A_1.$$

The self-consistency of the interaction now leads to the constraints

$$g^{\alpha}g^{\beta}g^{\alpha} = A_3 g^{\beta},$$

where A_3 is a constant, yet to be determined.

We note that these non-linear equations are invariant under a family of automorphisms. Consider the transformations:

$$g^{\alpha} \rightarrow U g^{\alpha} U^{\dagger}$$

$$g^{\alpha} \rightarrow V^{\alpha\beta} g^{\beta}; \;\; V^{\alpha\beta}V^{\alpha\gamma} = \delta^{\beta\gamma}$$

which leave the system of equations satisfied by g^{α} invariant. These correspond to linear transformations amongst the members of the nucleon doublet and (real) unitary transformations amongst the pions respectively. Hence in this simplified case these automorphisms preserve the self-consistency of the dynamics.

We could, in the present case, by virtue of these automorphisms, take g^1 and g^2 to be traceless and g^1 to be proportional to the first Pauli matrix so that

$$g^1 = G\tau^1$$

The trace normalization and the automorphisms can then be used to write

$$g^2 = G\tau^2$$

The trace normalization and the requirement that the sum of the squares of these three matrices be proportional to the unit matrix then gives the two alternatives

$$g^3 = \begin{cases} G\tau^3 \\ G\,\mathbf{1} \end{cases}.$$

The second solution is eliminated by the cubic constraint yielding the charge-independent coupling

$$g^{\alpha} = G\tau^{\alpha}.$$

It is interesting to observe that electric charge conservation has been deduced rather than assumed. We now compute A_3 to be $-G$.

As a second example, we consider the sigma hyperon-pion coupling. We proceed in a completely similar manner to deduce the equations

$$\mathrm{Tr}\,(g^\alpha g^\beta) = A_1 \delta^{\alpha\beta}$$

$$\sum_\alpha (g^\alpha g^\alpha) = A_2 \mathbf{1}$$

$$\sum_\alpha g^\alpha g^\beta g^\alpha = A_3 g^\beta$$

which are again invariant under the automorphisms

$$g^\alpha \to U g^\alpha U^\dagger$$

$$g^\alpha \to \sum_\beta V^{\alpha\beta} g^\beta; \qquad \sum_\alpha V^{\alpha\beta} V^{\alpha\gamma} = \delta^{\beta\gamma}$$

The matrices are now 3×3 hermitian and the constants A_1 and A_2 are equal. The solution however is not unique. To characterize the solutions more precisely we define two sets of 3×3 hermitian matrices satisfying these equations to be equivalent if they can be obtained from each other by suitable automorphisms. This equivalence relation defines equivalence classes of sets of matrices. There are three such classes:

(i) $[g^\alpha, g^\beta] = 0$

(ii) $[g^\alpha, g^\beta] = i\mu\,\epsilon^{\alpha\beta\gamma} g^\gamma$

(iii) $[g^\alpha, g^\beta] = \nu g^\gamma$, α, β, γ cyclic

The first one corresponds to an Abelian group, the second to the charge-independent coupling (isospin group) and the third to invariance under a discrete group (three-dimensional representation of the symmetric group S_4. If we supplemented these requirements by conservation of electric charge, only the charge-independent solution remains.

These considerations can be extended to more complicated, but physically relevant cases like the interaction between unitary multiplets. In these cases it appears necessary to use auxiliary constraints like isospin invariance and/or charge conjugation invariance to be able to deduce the symmetry. Two points may be noted however. First, the composite particle is thought of in the two-particle approximation, but the spatial part of the wave-function does not appear explicitly. Second, the basic consequence of the self-consistency requirements are algebraic non-linear relations whose primitive form is different from that of a Lie algebra. For want of a better name we may call the three-index entities satisfying these relations a symmetry algebra[4]. What is contained in the derivation of continuous symmetries is the establishment of a Lie algebra structure for a (suitably constrained) symmetry algebra.

If we have a self-consistent system which has an interaction structure invariant under a group, discrete or continuous, we can immediately define a set of multiplicative quantum numbers which are in general non-commutative and which preserve the group composition law: we simply make it act on each particle multiplet as if it were a free particle multiplet. If the group is a continuous group we can correspondingly define a Lie algebra ("current algebra" of dynamical variables.

The line of investigation above is satisfactory to a certain extent; it illustrates the possible origin of symmetry. But in practice symmetries are "broken". We must then search for a mechanism for the breaking of symmetries. One possibility is that these symmetries are being "masked" by a mechanism familiar to us in both classical and quantum mechanics. We discuss this masking mechanism in the next section. We note in passing that the two-particle approximation that we used is not necessarily adequate; and we could expect corrections due to this. Later we shall attempt to establish a connection between the masking of symmetries and these corrections.

IV. DYNAMICAL AUTOMORPHISMS AND THE APPARENT REDUCTION OF SYMMETRY

Consider a system of several free particles, assumed spinless for convenience. Then the momenta of each of the particles is conserved. What happens when the particles interact, say, by a known potential? The total momentum as well as the angular momentum in the centre-of-mass frame is conserved; the new total energy is also conserved. But where are the other constants of motion [13]?

To answer this question we consider a special class of potentials which allow no bound states and admit the usual kind of scattering states. In such a case the spectrum of the interacting Hamiltonian and that of the free Hamiltonian coincide; moreover the "eigenstates" of the interacting Hamiltonian expanded in terms of "eigenstates" of the free Hamiltonian furnish a set of continuous matrices which can be shown to be unitary. The matrix of these coefficients has the further property that it unitarily transforms the free Hamiltonian into the interacting Hamiltonian. Not all unitary transformations do lead to Hamiltonians which produce non-trivial scattering, but some do. We shall refer to these as strong unitary transformations. We now observe that the strong unitary transform of the "constants of motion" of the free Hamiltonian are "constants of motion" of the interacting Hamiltonian. Some of these, like the total momentum, are unaltered under this transformation; these may be called the regular constants of motion. Others are explicitly dependent on the structure of the interactions and are called the irregular constants of motion. The complicated expressions for the

[4] See footnote 1.

irregular constants as a function of the primitive dynamical variables makes it difficult to identify them in general; we have consequently an apparent "reduction of the symmetry" due to interaction.

The strong canonical transformations [14] are especially simple to work out for the familiar cases of the separable potential and the Lee model (in their lowest sectors). We outline the construction for the Lee model in the $N\theta$ sector in a "stripped-down" form in which the kinematical factors are all eliminated. This corresponds to choosing the S-wave amplitude expressed as a function of the energy and suitably normalized. The wave-functions correspond to components a and $b(\omega)$ which are respectively the amplitudes for the V particle and the $N\theta$ two-particle state with centre-of-mass energy ω; and these are normalized by

$$|a|^2 + \int_\mu^\infty d\omega\, |b(\omega)|^2 = 1.$$

The free Hamiltonian is represented by the matrix

$$H_0 = \begin{pmatrix} m & 0 \\ 0 & \omega\delta(\omega-\omega') \end{pmatrix},$$

and the interacting Hamiltonian by

$$H = H_0 + V \qquad V = \begin{pmatrix} 0 & f(\omega) \\ f^*(\omega') & 0 \end{pmatrix} + \begin{pmatrix} m_0 - m & 0 \\ 0 & 0 \end{pmatrix}$$

The last term on the right-hand side corresponds to mass renormalization. Since both H and H_0 have continuous spectrum they have only improper (non-normalizable) eigenvectors, with the exception of the single discrete state. To find the eigenstates of H we write

$$\begin{pmatrix} m_0 & f(\omega) \\ f^*(\omega') & \omega\delta(\omega'-\omega) \end{pmatrix} \begin{pmatrix} a_\lambda \\ b_\lambda(\omega) \end{pmatrix} = \lambda \begin{pmatrix} a_\lambda \\ b_\lambda(\omega') \end{pmatrix}$$

or, more explicitly:

$$(\lambda - m_0)\, a_\lambda = \int d\omega\, f(\omega) b_\lambda(\omega),$$

$$(\lambda - \omega') b_\lambda(\omega') = f^*(\omega') a_\lambda$$

Hence, apart from normalization

$$b_\lambda(\omega') = \delta(\lambda - \omega') + (\lambda - \omega' + i\epsilon)^{-1} f^*(\omega') a_\lambda$$

By direct substitution we obtain

$$a_\lambda = \frac{f(\lambda)}{\left[(\lambda - m_0) - \int \frac{|f(\omega)|^2 d\omega}{\lambda - \omega + i\epsilon}\right]},$$

$$b_\lambda(\omega') = \delta(\lambda - \omega') + \frac{f(\lambda)\, f^*(\omega')}{(\lambda - \omega' + i\epsilon)\left[(\lambda - m_0) - \int \frac{|f(\omega)|^2 d\omega}{\lambda - \omega + i\epsilon}\right]}.$$

There is also a solution for the discrete eigenvalue m, provided

$$m - \int \frac{|f(\omega)|^2 d\omega}{\lambda - m} = m_0$$

The solution in this case is

$$a_m \quad Z, \; b_m(\omega') = \frac{Z f^*(\omega')}{m - \omega'}.$$

Consequently, for normalization,

$$Z|^2 = \frac{1}{1 - \int \frac{|f(\omega)|^2 d\omega}{(m - \omega)^2}}$$

The components of these solutions a_λ, $b_\lambda(\omega')$, a_m, $b_m(\omega')$ constitute the components of a unitary matrix

$$U = \begin{pmatrix} Z & \frac{Z f^*(\omega')}{m - \omega'} \\ \frac{f(\lambda)}{\beta(\lambda + i\epsilon)} & \delta(\lambda - \omega) + \frac{f(\lambda)\, f^*(\omega)}{(\lambda - \omega + i\epsilon)\,(\lambda - m_0)\,\beta\,(\lambda + i\epsilon)} \end{pmatrix}$$

where

$$\beta(u) = u - m_0 - \int \frac{|f(\omega)|^2 d\omega}{u - \omega} = (u - m)\left\{1 - \int \frac{|f(\omega)|^2 d\omega}{(u - \omega)\,(\omega - m)}\right.$$

which has the property of unitarily transforming the free Hamiltonian into the interacting Hamiltonian according to

$$H = U^\dagger H_0 U.$$

It is obvious from the nature of these functions that they correspond to a non-trivial scattering.

Let us now consider a general strong unitary transformation. Dynamical variables which had a "simple" structure now acquire a more formidable appearance. However the algebraic relations amongst the dynamical variables remains unchanged. In particular, if they obeyed Lie algebra relations amongst themselves, the complicated-looking transformed dynamical variables also obey the same Lie algebra relations. However their matrix elements themselves will look quite different.

This situation is of course not entirely unfamiliar; the electromagnetic interaction of the nucleon, for example, is altered by the fact that it participates in strong interactions. If we treat the pion-nucleon interaction in terms of a suitable strong unitary transformation, we can compute the effective electromagnetic interaction. A similar comment applies also to the beta decay interaction. If it happens that there are some simple dynamical variables left invariant by the strong unitary transformation, like the electric charge, this would imply corresponding constraints on the effective interactions of the interacting particles.

MASKED SYMMETRIES

Let us return to the dynamical symmetries derived from self-consistency requirements. We have seen how in some cases the symmetry algebra reduces to a Lie algebra. But both these could be transformed under suitable automorphisms. The symmetry group transformed one-particle states into one-particle states and so did the automorphisms. (In all cases studied so far, the automorphisms belong to the enveloping algebra, and so this was to be expected.) Experimentally, however, these symmetries are only approximate.

We now propose that these symmetries are "masked" i.e. they undergo a strong unitary transformation, which does not, in general, leave one-particle states invariant. If we consider the actual Lie algebra ("current algebra") it is a masked symmetry: the one-particle-to-one-particle matrix elements of the "currents" do not exactly satisfy the Lie algebra structure. They should be supplemented by the contribution of many-particle intermediate states. Let us consider the entire interaction to be brought about by a strong unitary transformation. (There is no reason why it should not be!) In this case for the free particle system there is a basic Lie algebra, namely the one which is used for classifying the particle multiplets as irreducible representations. This Lie algebra becomes masked by the strong unitary transformation. If we try to work with the particle-to-particle matrix elements only, the masked symmetry becomes a broken symmetry. It is again worth pointing out that no requirement of equal masses of members of a multiplet is dictated at any stage.

Not all strong unitary transformations necessarily mask a symmetry. If we consider the charge-independent interaction between pions, nucleons, nucleon isobars and vector mesons, only part of the SU(4) Lie algebra becomes masked. The isospin structure is unaltered but the spin-dependent part of the SU(4) structure is masked. Similar comments apply to the higher groups like SU(6) and U(6) $\times$ U(6) etc.

The possibility of considering strong unitary transformations as dynamical automorphisms shows, incidentally, that a purely self-consistent approach to the determination of current algebras cannot lead to a unique solution. For, given any one such solution we could always get a different, but isomorphic, current algebra by a strong unitary transformation. While the mathematical structure of both realizations of the current algebra is the same, physically they are quite different; for example, they predict different photo-production amplitudes. The apparently unique results "reached" in some self-consistent calculations [15] must therefore be due either to the nature of the approximations or to additional (hidden) dynamical assumptions made in the course of these calculations.

VI. MAGNITUDE OF THE GAMOW-TELLER COUPLING CONSTANT

As an application of these ideas, let us consider the renormalization of the axial vector coupling constant in nuclear beta-decay (the Gamow-Teller coupling constant). We shall consider a strong unitary transformation which mixes the nucleon with nucleon-pion states in a charge-independent fashion; for simplicity we restrict ourselves to the two-particle approximation. If we go to the centre-of-mass frame, the nucleon is in a $\frac{1}{2}^+$ state and the pion-nucleon system is in a $^2P_{\frac{1}{2}}$ state with a centre-of-mass (total) energy M. The strong unitary transformation as applied to the nucleon state can now be written

$$|N\rangle \to Z|N\rangle + \int dM\, \phi(M)|M\rangle$$

with the normalization condition

$$|Z|^2 + \int dM\, |\phi(M)|^2 = 1.$$

If we now define an axial vector interaction $A_{\mu+}$ which is represented by $G_1\gamma_\mu\gamma_5\tau_+$ between the $|N\rangle$ states, after transformation its matrix elements are more complicated. The nucleon to nucleon matrix element (for nucleons in the centre-of-mass frame) is given by

$$\langle N|A_{\mu+}|N\rangle = \langle N|\mathscr{P}_{\text{II}} A_{\mu+} \mathscr{P}_{\text{II}}|N\rangle$$

$$= G_1 |Z|^2 \bar{u}_m \gamma_\mu \gamma_5 \tau_+ u_m + \int dM\, |\phi(M)|^2 \frac{G_1}{9}\, \bar{u}_M \gamma_\mu \gamma_5 \tau_+ u_M$$

$$\simeq \left\{1 - \frac{8}{9}(1 - |Z|^2)\right\} G_1 \bar{u}_m \gamma_\mu \gamma_5 \tau_+ u_m,$$

where $\mathscr{P}$ is the projection operator to the spin half, isotopic spin half, even parity states. The factor (1/9) comes from the nucleon transition matrix elements for such pion-nucleon states.

We now compute the nucleon to nucleon-pion matrix element of $\mathscr{P} A_{\mu+} \mathscr{P}$ by a similar method to obtain

$$\langle N | A_{\mu+} \mathscr{P}_{\shortparallel} | M \rangle = G_1 Z^* Z_M \bar{u}_m \gamma_\mu \gamma_5 \tau_+ u_m$$

$$\frac{1}{8} G_1 \int dM' \phi(M) \phi_M(M') \bar{u}_M \gamma_\mu \gamma_5 \tau_+ u_M$$

$$\simeq \frac{8}{9} G_1 Z^* \phi(M) \bar{u}_m \gamma_\mu \gamma_5 \tau_+ u_m$$

In the course of the calculation we have used the coefficients of the transformation

$$|M'\rangle \to Z_M |N\rangle + \int d M' \phi_M(M') |M'\rangle$$

and the unitary property of the transformation. We note that the effective (nucleon to nucleon) Gamow-Teller coupling constant is

$$G_A = \left\{ 1 - \frac{8}{9} (1 - |Z|^2) \right\} G$$

We can estimate the numerical value of $|Z|^2$ if we (physically!) identify the matrix element

$$\langle N | A_{\mu+} | M \rangle$$

with the pion-nucleon scattering amplitude (off the mass-shell for one of the pions) apart from a scale factor. Note that in view of the assumption that the strong unitary transformation is charge-independent the appropriate pion-nucleon scattering amplitude is that for the $I = \frac{1}{2}$ $^2P_{\frac{1}{2}}$ state (i.e. the amplitude related to the phase shift δ_{11}) and is thus distinct from other sum rules involving the axial vector coupling constant [16]. The physical assumption required is the divergence relation

$$-i \partial^\mu A_{\mu+} = C \phi_{\pi+} = C(m_\pi^2 - \Box^2)^{-1} j_{\pi+},$$

where ϕ_π is the pion field and j_π is its (pseudoscalar) source.

For small momentum transfers we can write, for the matrix element of the divergence of the axial vector current between the $^2S_{\frac{1}{2}}$ one-nucleon state and the $^2P_{\frac{1}{2}}$ pion-nucleon state, the expression

$$\langle N | -i \partial^\mu A_{\mu+} \mathscr{P}_{\shortparallel} | M \rangle = \frac{8}{9} \phi(M) Z^* G_1 \bar{u}_m (-i \overleftrightarrow{\partial}^\mu \gamma_\mu \gamma_5 \tau_+ u_M$$

$$= \frac{8}{9} \phi(M) Z^* G_1 (m + M) \bar{u}_m \gamma_5 \tau_+ u_M$$

$$= \frac{8}{9} \frac{C}{m_\pi^2} \langle N | j_{\pi+} \mathscr{P}_{\shortparallel} | M \rangle,$$

since:

$$\langle N | A_{\mu+} \mathscr{P}_{11} | M \rangle \simeq \frac{8}{9} Z^* G_1 Z_M \bar{u}_m \gamma_5 \gamma_\mu \tau_+ u_M$$

The quantity $\langle N | j_{\pi+} \mathscr{P}_{11} | M \rangle$ can be evaluated from pion-nucleon scattering data by suitable off the mass-shell extrapolation and this enables us to determine $\phi(M)$, given the proportionality constant C. The constant C is evaluated by normalizing the one-nucleon matrix elements of the divergence relation (for small momentum transfer)

$$\langle N | -i\partial^\mu A_{\mu+} | N \rangle = 2m \left\{ 1 - \frac{8}{9}(1 - |Z|^2) \right\} G_1 \bar{u}_m \gamma_5 \tau_+ u_M$$

$$= \frac{C}{m_\pi^2} \langle N | j_{\pi+} | N \rangle = \frac{C}{m_\pi^2} \sqrt{2}\, g\, \eta\, \bar{u}_m \gamma_5 \tau_+ u_m,$$

where g is the charge-independent neutral pion-proton coupling constant (and η is a factor to allow for the fact that usually we define the coupling constant with the pion on the mass-shell). This gives

$$C = \sqrt{2}\, m\, m_\pi \left\{ 1 - \frac{8}{9}(1 - Z|^2) \right\} (G_1 | g\eta).$$

(The actual evaluation of the expected correction obviously needs a great deal of work!)

There are two remarks that can be made: First, since $|Z|^2 \lesssim 1$ the effective axial vector constant is reduced from its original value; this is to be contrasted with the vector coupling constant which is unaffected. Second, the magnitude of the renormalization is expected to be small since the pion-nucleon scattering in the (1, 1) channel is small.

The SU(4) or SU(6) theory (or their covariant counter-part) gives for the primitive axial vector coupling constant,

$$G_1 = -\frac{5}{3} G_V.$$

With the observed value of

$$\left\{ 1 - \frac{8}{9}(1 - |Z|^2) \right\} G_1 = 1.2\, G,$$

this implies:

$$z|^2 = 1 - \int |\phi(M)|^2 dM = 0.68.$$

It is possible to derive, by a simple modification of the above derivation, a sum rule relating the renormalization of the nucleon to nucleon isobar axial vector coupling with (the off the mass-shell extrapolation of) the (3, 3) pion-nucleon scattering amplitude.

VII. THE PROBLEM OF COVARIANT SYMMETRIES

Let us return to the notion of masked symmetries and view the nature of the covariant spin-dependent symmetry schemes that have been discussed recently. Treated as conventional symmetries, they are not only badly broken but are even intrinsically broken. The covariant symmetries of the $\tilde{U}(12)$ and $\tilde{U}(8)$ type are not even valid for "free fields" since they certainly violate the free field commutation relations. The possibility of viewing them as masked symmetries must then presuppose an even more primitive dynamical level. Some speculations in this direction are made below.

Consider a local covariant field (quark field) which, for convenience, is taken as a spinor field furnishing the fundamental representation of a suitable non-compact unitary group. For the $\tilde{U}(8)$ and $\tilde{U}(12)$ theories these groups are respectively SU(4, 4) and SU(6, 6). These fields are not associated with any particles a priori. They are however strongly coupled to each other, not necessarily locally, but in a manner which allows independent "unitary" transformations on the indices. The (quasi-local) excitations of the field furnish representations of these "unitary" groups, which are however reducible. They are by no means the lowest-lying levels. To obtain the low-lying levels we have to form coherent combinations of these excited states. If they are to furnish unitary representations of the Poincaré group, they should also be able to support particle-like excitations and these are the irreducible representations belonging to the point spectrum of the (squared) mass operator. The formation of particles by itself does not violate the symmetry associated with the index transformation group; what does break this symmetry is the realization of only one kind of particle with a fixed mass. This is very similar to the physical phenomenon found with a variety of systems, like the feromagnet; for this case, the primitive structure is invariant under rotations, but the ground state is not invariant under the same group. While for a finite ferromagnet one could go from one orientation to another by a rotation, for an infinite (or, in practice, a macroscopic) ferromagnet such rotations are not physically allowed. The same thing happens here also. Microscopic coherence of excitations can be transformed into another one by the index transformation group. But any long-range order, like the infinite length of coherence for a free particle, forbids the index transformation group from acting on the state. The new states so obtained do not have the full symmetry of the index transformation group.

A residual spin-dependent relativistic group still applies. This is the group of relativistic spin (FW spin) transformations [17] on the particles. By suitably defining these relativistic (FW spin) transformations for the negative energy states also one derives the W-spin transformations [18] which are easier for computations. The use of W-spin in particle reactions where it can be applied in a relativistically invariant manner has been discussed elsewhere in these Proceedings [19].

One could, in the search for further kinematic transformation groups (which could then be candidates for being "masked"!) exploit the known particle-antiparticle symmetry together with the notion of FW-spin transformations to generate particle-antiparticle algebras. Speculations concerning their possible uses have been presented elsewhere [20].

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